The Twentieth Annual Nebraska Conference for Undergraduate Women in Mathematics

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TALK ABSTRACTS

PLENARY TALKS

Dr. Jill Pipher
Professor of Mathematics
Brown University
Mathematical Ideas in Public Key Cryptography

The concept of public key encryption was introduced in the famous 1976 paper "New Directions in Cryptography" by Diffie and Hellman. Within a couple of years, Rivest, Shamir and Adelman produced the first published example of a public key cryptosystem, and RSA encryption is still widely used for secure communication. In this lecture, I will give some historical background to encryption, both private and public key, and explain some of the mathematical ideas involved in several different encryption systems. In particular, we'll focus on lattice-based encryption schemes, like NTRU, an efficient public key system due to Hoffstein, Silverman and myself, that continues to remain secure against quantum attacks.

Dr. Irena Swanson
Professor of Mathematics
Reed College
Life in the Algebra Lane

Algebraists solve equations systematically and in the process build relevant structures and theorems. As a starting point we will take equations from linear algebra and look at them with Hilbert's Nullstellensatz, Grete Hermann's bounds, and Gröbner bases.

Talks by Undergraduate Students

Chloe Avery, University of California, Santa Barbara
Caitlyn Booms, University of Notre Dame
How Noncatenary Can a UFD Be?

Let A be a local (Noetherian) integral domain with unique maximal ideal M. In algebra, nearly every nice A that mathematicians encounter is catenary. However, we say that A is noncatenary if its prime ideal structure satisfies a strange condition. In 1946, Cohen conjectured that every local integral domain is catenary. However, in 1956, Nagata gave an example of a noncatenary local integral domain, and he then conjectured that all local unique factorization domains (UFDs) are catenary. Once again, this was proven false by Heitmann in 1993 who gave an example of a noncatenary UFD. In addition, Heitmann proved in 1979 that there is no finite bound on how noncatenary a local integral domain can be. Using theorems proven during our research program this summer, we have been able to extend this result to show that there is no finite bound on how noncatenary a local UFD can be. We'll briefly discuss our theorems and give an example that demonstrates this result.

Carissa Babcock, Alverno College
Competition and $(1, 2)$-Step Competition Numbers in 4-Cycle Graph Variations

In 2013, Factor, Merz and Sano asked the question: Are there graphs other than C_4, the cycle on 4 vertices, where the competition number of a graph G, denoted $k(G)$, is greater than its $(1, 2)$-step competition number, $k(1, 2)(G)$? The competition number $k(G)$ is defined as the smallest nonnegative integer k such that a graph G with k isolated vertices is the competition graph for some acyclic digraph. Similarly, the $(1, 2)$-step competition number is defined as the smallest nonnegative integer k such that a graph G with k isolated vertices is the $(1, 2)$-step competition graph for some acyclic digraph D. An (i, j)-step competition graph is the generalized concept in which $\{x, y\}$ will be an edge in the (i, m)-step competition graph, denoted $C(i, m)(D)$, if for some $z \in V(D) - \{x, y\}$, $d_{D-y}(x, z) \leq 1$ and $d_{D-x}(y, z) \leq 2$ or $d_{D-x}(y, z) \leq 1$ and $d_{D-y}(x, z) \leq 2$. Here we explore classes of graphs that provide a partial answer to the question asked in that paper and prove that the relationship exists.

Bryn Balls-Barker, Brigham Young University
Optimization of Spectral Algorithms for Link Prediction

The question of which edges will most likely be added next to a network is commonly known as the link prediction problem. Many approaches to solving this problem have been proposed including the use of effective resistance, which can be implemented using eigenvalues and eigenvectors of the Laplacian matrix of a graph. Various methods exist for obtaining these spectral properties. This step accounts for the majority of the temporal complexity of our algorithm to compute effective resistance. We seek to find an efficient method that minimizes this complexity while maintaining accuracy.

Maya Banks, Carleton College
A model of hierarchy emergence in complex networks

Given a directed network, we can infer a hierarchy by extracting a ranking of the vertices using the adjacency matrix of the network. In systems such as animal social groups or faculty hiring networks, these rankings allow us to better understand the relative status of different individuals. While there exist different ranking models that allow us to observe hierarchy in established systems, we are left with many questions about how hierarchical structure evolves in networks in the first place. In real networks where the edge set changes over time, we expect that hierarchy in the network affects the formation of new edges. At the same time, the hierarchy that we observe is extracted from the edges currently present in the network. We explore a dynamical model of hierarchy emergence in networks that produces a strongly tiered hierarchical structure, using a physics-inspired method of ranking vertices.

Hannah Barta, Central Washington University
Study of Vortex Dynamics with Free Surface Flow

We examine the deformation of varying shapes of collections of point vortices beneath a free surface of an inviscid fluid with a flat bottom boundary. Initial simulations investigate the deformation of vortex sheets in a manner which resembles the Kelvin-Helmholtz instability. With large numbers of point vortices, the simulations we run show that these sheets tend to deform into elliptical patches. Several other shapes of point vortex arrays are simulated, and almost all deform into an elliptical shape. Upon deciding to simulate an ellipse as a starting shape, we observe that an initial elliptical shape deforms the least with time. To increase the realism of these simulations, we introduce a mollified kernel, which slows the speeds of vortices that are close together and yields ellipses that retain their shape with time. We introduce a metric to measure the deformation of these ellipses before and after mollification.

Joanne Beckford, Williams College
Tina Rajabi, Edmonds Community College
The Combinatorics of Factorial Base Representation

Every non-negative integer can be written using what is known as the factorial base representation. We define this notion and explore certain combinatorial structures arising from the arithmetic of these representations. In particular, we will investigate the sum-of-digits function, carry sequences, and a partial order referred to as digital dominance. Finally, we describe an analog of a classical theorem due to Kummer that relates the combinatorial objects of interest by constructing a variety of new integer sequences.

Emalina Bidari, Northern Arizona University
Structure of braid graphs for reduced words in Coxeter groups

Every element w of a Coxeter group W can be written as an expression in the generators, and if the number of generators in an expression (including multiplicity) is minimal, we say that the expression is reduced. Every pair of reduced expressions for the same group element are related by a sequence of commutations and so-called braid moves. We say that two reduced expressions are braid equivalent if they are related by a sequence of braid moves. Given a reduced expression r for a group element w, we can form the corresponding braid graph whose vertices are the reduced expressions that are braid equivalent to r and two reduced expressions are connected by an edge if there is a single braid move that converts one reduced expression into the other. In this talk, we will describe the overall structure of braid graphs for reduced expressions in Coxeter groups of type A and B in terms of products of graphs.

Caitlyn Booms, University of Notre Dame
see **Chloe Avery**

Rachel Chaiser, University of Puget Sound
The Combinatorics of Zeckendorf Representations

In this talk we acknowledge the idea that natural numbers exist regardless of their representation. We explore the properties of a representation that utilizes the Fibonacci sequence as the base, called the Zeckendorf representation. We examine the combinatorics arising from the arithmetic of these representations, with a particular emphasis on understanding the "Zeckendorf tree" that encodes them. We introduce several new results related to the tree, allowing us to develop a partial analog to Kummer's classical theorem for counting the number of carries when adding in a prime base.

Jacquelyn Chapman, Youngstown State University
Julie Phillis, Youngstown State University
A Mathematical Simulation of Bone Cell Behavior: Osteoclasts

The primary function of bones involves three different cells: osteoclasts, osteoblasts, and osteocytes. Osteoclasts are large bone cells that absorb bone tissue, osteoblasts are cells that create new bone, and osteocytes are essentially osteoblasts that have been trapped in the secreted bone matrix. All three cells work together concurrently and are responsible for what the bone is able to do on a daily basis to maintain homeostasis. This talk focuses on one of these three cells, osteoclasts. The research done was in collaboration with the Biomedical Engineering Department at The University of Akron, who provided the lab work. A mathematical model was developed in order to simulate the behavior of these cells in the lab.

Malori Cloys, Austin Peay State University
Neighborhood-Prime Labelings of Tree Graphs

A neighborhood-prime labeling of a graph is a variation of a prime labeling in which the vertices are assigned labels from 1 to $|V(G)|$ such that the greatest common divisor of the neighborhood of each non-degree 1 vertex is equal to 1. In this talk, we examine neighborhood-prime labelings of several types of tree graphs, such as caterpillars, firecrackers, spiders, banana trees, and complete binary trees. These labelings support the theory that all tree graphs have a neighborhood-prime labeling.

Olivia Conway, University of Oklahoma
Effects of a Vaccine-Adverse Minority on Vaccination Dynamics

Population vaccination strategies can be modeled as a public goods game, in which some free-riders attempt to benefit from the herd immunity generated by their vaccinated peers. This research takes a game-theoretic approach to represent how vaccine compliance evolves when a proportion of the population is predisposed to distrust vaccines. We further demonstrate the influence of minority opinion in structured neighborhoods. A two-dimensional cellular automaton model superimposes a lattice-like social network to illustrate the spread of contagion and observe the effect of peer networks on social learning.

Lydia DeMorett, College of St. Benedict/St. John's University
Modeling Assortative Mating: Creating Configurations of Conflicting Counterparts

Studies show that individuals display preferences when choosing a mate. Assortative mating models take into account the organisms' preferences versus a random mating model which, while computationally easier, is less realistic. This study uses Markov chains to create a flexible model to analyze long term effects of different types of assortative mating. Previous models considered strictly positive or strictly negative assortative mating. We investigate when positive and negative assortative mating are combined.

Olga Dorabiala, Pennsylvania State University
Equation of State for Matter at Extreme Conditions

We provide a new statistical model, the Saha-Hypernetted-Chain model, for computing certain state variables of a plasma, including pressure and charge moments. The model is designed to be less computationally expensive than molecular-level simulations but more accurate than existing statistical models. Such a model is of interest to the Lawrence Livermore National Laboratory because the large-scale fluid dynamics computations performed there rely on the calculation of plasma state variables (at every point in a discretization of space) as a fundamental subroutine. Our approach is to combine the Saha model for computing ionization level proportions with the hypernetted-chain (HNC) model for computing pair-correlation functions. This approach addresses the main draw-back of previous statistical methods that use the Saha model: a poor estimation for the shift in ionization energy due to many-body effects. This shift can be more accurately estimated using the pair-correlation functions given by the HNC model.

Callie Duque, South Dakota State University
Extending the Bôcher-Grace Theorem to Quadrilaterals and Other Polygons

In recent years several published papers have investigated ellipses inscribed in triangles in the complex plane and related the geometry of the ellipse to a polynomial whose zeros are the complex coordinates of the vertices of the triangle. The foci of the ellipse are then the zeros of the derivative of this polynomial. In 2008, the result was revisited and called 'the most marvelous theorem in mathematics'. We call this the Bôcher-Grace Theorem after two early authors who discussed this result. The purpose of this undergraduate research project is to extend this type of result to quadrilaterals and other polygons. Since then several papers have been published with alternate proofs and extensions of the result to quadrilaterals and other polygons. This research has opened up more ideas within the fields of complex numbers and geometry. Therefore, we will more thoroughly understand the interworking of geometry and how different shapes and figures interact.

Alexandra Embry, Indiana University
kth-Order Fibonacci-like Polynomials

The well-studied Fibonacci polynomials are described by $F_n(x) = xF_{n-1}(x)+F_{n-2}(x)$ with $F_0 = 1$ and $F_1 = x$. We extend this recursive polynomial sequence to $G_n^{(k)}(x) = xG_{n-1}^{(k)}(x) + G_{n-k}^{(k)}(x)$ and further to $H_n^{(k)}(x) = xH_{n-1}^{(k)}(x) - H_{n-k}^{(k)}(x)$ with initial conditions $G_0^{(k)} = G_1^{(k)} = ... = G_{k-1}^{(k)} = 1$ and $H_0^{(k)} = H_1^{(k)} = ... = H_{k-1}^{(k)} = 1$. When $k = 2$, the $G_n^{(k)}$ polynomials are the Fibonacci polynomials with altered initial conditions. When $k = 3$ and $x = 1$, the $G_n^{(k)}$ polynomials describe Narayanaś Cow Sequence. In this talk, we present a closed form for these polynomials from which we derive a class of significant integer sequences. We also study some interesting analytic properties of these polynomials, especially concerning the behavior of their roots. We establish that the minimal real roots of $G_n^{(k)}(x)$ converge uniformly to some number $-3 < r^{(k)} < -1$, and the maximal real roots of $H_n^{(k)}(x)$ converge uniformly to 2. MATLAB simulations show that the roots of $G_n^{(3)}$ are rational for only $n = 3, 5, 6, 10$ and 19, and those of $H_n^{(3)}$ are rational for only $n = 3$ and 12. Computer-assisted curve fitting using MATLAB suggests that the relative rates of convergence of the minimal real roots of $G_n^{(k)}$ and the maximal real roots of $H_n^{(k)}$ are of exponential order.

Kelly Emmrich, University of Wisconsin-La Crosse
Norm-Euclidean Ideal Classes in Galois Cubic Fields

Lenstra introduced the notion of a norm-Euclidean ideal class as a generalization of norm-Euclideanity of a number field. He classified all quadratic number fields possessing a norm-Euclidean ideal class. We investigate the Galois cubic case. We show that up to discriminant 10^{11} at most two such number fields possess a nontrivial norm-Euclidean ideal class, and we conjecture no more exist. In an attempt to settle our conjecture, we prove explicit bounds on the first few non-residues of cubic characters under the generalized Riemann hypothesis.

Emma Fancher, University of North Alabama
Coding for Distributive Storage from Curves

Historically, researchers have predominantly looked at Hermitian codes for use in distributed storage. Algebraic geometric codes (AG-codes) have also been studied, and the generation algorithm of such codes is known. Because AG-codes have many advantages over Hermitian codes, our goal was to extend beyond the Hermitian case to explore the idea of using generalized AG-codes, specifically in the context of distributed storage. We investigated their structure and compared these codes to the Hermitian case, approximating the maximum number of total correctable errors. Based on our discoveries about AG-codes, we expanded and improved an already existing decoding algorithm (which may enable decoding of an otherwise non-decodable code) while exploring what occurs if certain pieces of information cannot be retrieved.

Nina Galanter, Grinnell College
Tiffany Jann, University of California, Berkeley
Efficient Communication in Distributed Machine Learning

To train large-scale machine learning models more quickly, researchers have adapted model optimization algorithms for distributed environments, where communication between machines can be costly. Increasing communication frequency increases communicating cost, but improves convergence rate. Microsoft Research Asia, our project sponsor, is interested in optimizing communication in distributed machine learning. We investigate the effect of communication frequency on the convergence of distributed Stochastic Gradient Descent (SGD) through theoretical and experimental approaches. We conduct convergence analysis on convex models for synchronous and asynchronous distributed SGD. We perform synchronous and asynchronous distributed SGD experiments with the Logistic Regression, Convolutional Neural Networks, and Long Short Term Memory Neural Networks models. For each model, we vary communication frequencies and record the wall clock time and number of iterations SGD takes to converge.

Courtney Gale, Furman University
Using Social Media Analytics to Predict College Basketball Tournament Upsets

Betweenness is a measure of centrality for graphs, originally developed to analyze social networks. Applying this concept to college basketball, we used betweenness to identify "chaotic killers" in the NCAA Tournament: underdog teams whose inconsistent gameplay could either result in them flaming out in early rounds of the tournament or making a deep run. We calculated betweenness for each Division-1 college basketball team by creating a network of all 351 teams, with directed edges representing a game between two teams and the direction indicating victory. Teams with high betweenness were those that played inconsistently during the regular season. Upsets, particularly non-conference upsets, were emphasized by the measure, and we hypothesized that high-betweenness teams would also play inconsistently in the tournament. We found evidence to support this: teams seeded 7-16 from 2007 to 2017 in the top half of betweenness had a 36.11% higher variance in their number of tournament wins than low-betweenness teams with the same seeds. Furthermore, high-betweenness teams among the same seed group were also more successful in the NCAA tournament. Among teams identified as having at least a 20% chance of upsetting a typical lower seed, those with high betweenness averaged nearly double the number of tournament wins as those with lower betweenness.

Stephanie Garcia, Central Washington University
Breaking the Enigma

During World War II the Enigma machines, a rotor-based cipher machine, became widely used by military and government services to protect military and diplomatic communication. In 1995, Gillogly investigated an algorithm to break, or cryptanalyze, this cipher. The algorithm uses a numerical calculation, called the index of coincidence, to approximate how close our deciphered text is to standard English. We implemented this algorithm using Python and PyEnigma, a Py 3 library for simulating Enigma machines. We will describe the algorithm and then provide results on how successful it was.

Paige Graves, University of La Verne
Jessie Loucks, California State University, Sacramento
Hilbert Series of Certain Families of Numerical Semigroups

We say a subset S of the nonnegative integers is a numerical semigroup if S is closed under addition, S has a finite complement in the nonnegative integers, and 0 is an element of S. For a numerical semigroup S, the Hilbert series of S is given by the power series $H(S;t)$, which equals the sum over t^n, where n is an element of S. This series has a known rational form which can be found by using connections between numerical semigroups and algebraic topology. In the talk, we will discuss numerical semigroups and introduce some topology with the purpose of describing the rational form of the Hilbert series. We will also discuss and characterize some particular forms that $H(S;t)$ can take when S falls into certain classes of numerical semigroups.

Jiajing Guan, George Mason University
Variational Data Assimilation for Neuronal Network Models

The large size of neuronal networks presents a challenge to understanding the behavior of the whole network. It is therefore of utmost importance to reconstruct the behavior of the network using the measurements of only a few neurons. In this research, we use coupled Fitzhugh-Nagumo models to simulate a chaotic neuronal network. Through the Levenberg-Marquardt method, we assimilate the behavior of a complete dynamical network given the data of a few observable nodes in the network, and assign each unobservable node a condition number describing the difficulty of reconstructing the node dynamics.

Xiaotong Gui, Pomona College
Modeling of Ca2+ Dynamics during Fertilization in Caenorhabditis elegans worms

Fertilization is a strikingly complex process that involves multiple chemical and mechanical signals. Travelling calcium waves across fertilized oocytes have emerged as a critical signal for proper embryonic development in most animals. In this talk, we describe how we combine mathematical modeling and experiments to study Calcium travelling waves in Caenorhabditis elegans (C. elegans), where recent experimental findings have proposed that a local Ca2+ rise at the point of oocyte/sperm contact is followed by a global Ca2+ wave. Since the exact mechanisms of wave initiation and propagation are not well understood in this organism, we discuss how a bistable partial differentiation equation model can be used to explain the C. elegans calcium dynamics machinery. Our model is constructed to fit the observed traveling wave pattern and different wave initiation hypotheses are tested with simulations and compared with data.

Marcela Gutierrez, Northern Arizona University
Viola McCarty, Northern Arizona University
An Infinite Tree of Primitive Pythagorean Quadruples

A primitive Pythagorean quadruple (PPQ) is a positive integer 4-tuple (d, a, b, c) having no common nontrivial divisors that satisfies $d^2 = a^2 + b^2 + c^2$. We will present a construction of a complete infinite tree of PPQs resulting from the orbit of $(1, 0, 0, 1)$ under the action of a small family of matrices.

Emma Holzbach, St. Catherine University
Autumn Mortenson, St. Catherine University
θ-Twisted Involution Graphs on the Alternating Group

Extended symmetric spaces, which are $R_\theta = \{g \in S_n \mid \theta(g) = g^{-1}\}$ where θ is an involution, are a point of interest in algebraic geometry. We studied an operation using generating sets of the alternating group on the extended symmetric space. With this operation, we produced graphs and examined their properties. We will discuss characterizations of the regularity and the edge type associated with each closed walk of these graphs, along with open questions pertaining to the vertex sets.

Tiffany Jann, University of California, Berkeley
see **Nina Galanter**

Hye Kyung Kim, George Mason University
Dynamics of Red Coral Populations

Coral is an animal that plays an important role in the marine ecosystem. Consequently, a number of researchers have studied questions of conservation of the red coral population by introducing a data-based and high-dimensional discrete model. Numerical simulations of this model have given some first insights into the effects of changes in the mortality rate, as well as the effects of overfishing. We analyze their discrete time model for red coral populations to shed light on the long-term dynamics of the population. The model exhibits both fixed points and a Hopf bifurcation, as a function of the basic reproductive number. We demonstrate that after the Hopf bifurcation, neighborhoods of the fixed points converge to closed curves, which in turn approach extinction in certain parameter regimes. Furthermore, the numerical results have been verified through computer assisted proofs as a first step toward rigorous mathematical results.

Catherine King, University of Texas at Tyler
On λ-Unimodal Permutations

A specific class of permutations, called λ-unimodal permutations, have recently appeared in the study of characters of the symmetric group. A permutation is unimodal if, when written in one-line notation, the permutation is increasing then decreasing. A λ-unimodal permutation in $\mathcal{S}_n$ is a permutation comprised of k contiguous unimodal segments, where the lengths of the segments are determined by a composition $\lambda = (\lambda_1, \ldots, \lambda_k)$. In this talk, we will discuss permutation statistics, including fixed points and descents, on λ-unimodal permutations.

Rachel Koch, James Madison University
Metrics of Gerrymandering

A metric for determining the extent of political gerrymandering, called the Efficiency Gap (EG), has faced the Supreme Court this year. We will analyze EG's strengths, weaknesses, and the challenges it may face when it is scrutinized by the Court. We will also examine past court rulings pertaining to partisan gerrymandering and the various geometric and algebraic metrics which these have spurred. Finally, we present some initial findings for a new metric, the power of a vote, which focuses on the political power of the individual instead of the political power of the party. The power of a vote also works to quantify intent in redistricting decisions, an element missing from all previous metrics.

Neeraja Kulkarni, Carleton College
Factorizations of k-Nonnegative Matrices

A matrix is said to be totally nonnegative if all square submatrices in it are nonnegative. Factorizations of totally nonnegative matrices have long attracted attention because of their applications in combinatorics, dynamics and probability, as well as their interesting topological structure. In particular, the semigroup of invertible totally nonnegative matrices can be partitioned, based on their factorizations, into cells that form a regular CW-complex. The closure poset of this CW-complex is described by the Bruhat order on subwords of factorizations. Our work considers k-nonnegative matrices, in which all minors of order at most k are nonnegative. We give a minimal set of generators in two special cases: first, when $k = n - 1$ for any invertible matrices and second, when $k = n - 2$ for triangular matrices with 1's on the diagonal. We describe how these two semigroups of matrices can also be partitioned into cells, to which we extend the Bruhat order by describing the subwords that arise from our new factorizations. The resulting partial ordering on subwords is the same as the poset induced by the topological closure of the cells of k-nonnegative matrices in these cases. This work was done with Ewin Tang of U Texas, Austin and Joe Suk of Stony Brook University.

Jessica Linton, Benedictine College
Modeling Biological Invasion with the Reaction-diffusion Equation

Every year, invasive species cause irreversible damage to economies and ecosystems worldwide. Preventing the spread of such species is an important step toward reducing impact on native flora and fauna, along with preserving local economies. A noteworthy example is Japanese knotweed, *Fallopia japonica*, a perennial native to Eastern Asia. It was introduced to the United States in the 1870s as an ornamental plant and has since displaced native vegetation and clogged rivers. Since fragments from the main plant can generate new sprouts, transport of such fragments by river networks may play a key role in its spread. To better understand the impact of a river on the spread of Japanese knotweed, we applied the Crank-Nicolson time splitting method to a reaction-diffusion model and compared our results with field data to assess its accuracy.

Jessie Loucks, California State University, Sacramento
see **Paige Graves**

Britney Mazzetta, Ithaca College
Predicting U.S. Childhood Obesity through Mathematical Modeling

In the past 50 years, observational studies have shown obesity to be the most prevalent nutritional-based disease in the most affluent countries of the world, including the United States. A variety of diseases, which results in a higher mortality rate, have been linked to those who suffer with obesity. This link raises concerns and has led to an increase in preventative efforts to reduce this rate through several public health programs in and out of schools to, hopefully, reverse the epidemic trend. We will provide a unique interpretation to how the trend can be modeled by focusing on the social and nonsocial factors that influence obesity and the degree to which those factors influence people by applying United States' data to a new mathematical model. This model will adopt similar strategies that were developed by Lucas Jódar and his colleagues ("Modeling dynamics of infant obesity in the region of Valencia, Spain," 2008). Interpretations of population changes in regards to obesity trends can then be made. Through these models, predictions can also be formed that will identify future childhood obesity trends in the U. S. This rate can then be used in a predictive manner. Additional solutions on how childhood obesity trends in the U.S. can be reduced could also be concluded.

Viola McCarty, Northern Arizona University
see **Marcela Gutierrez**

Martha Morrise, Brigham Young University
Using Bayesian Inference to Reconstruct a Tsunami

An inverse problem is a situation in science which uses the results of a system to predict the conditions which caused them. A specific example of this is using Bayesian inference to reconstruct the location and conditions of earthquakes that occurred centuries ago, for which only anecdotal evidence exists. We use Markov chain Monte Carlo methods to find source parameters for earthquakes and tsunamis that occurred in Indonesia.

Autumn Mortenson, St. Catherine University
see **Emma Holzbach**

Kaela Nelson, Brigham Young University
What's in the Air? Using Mathematical Models to Predict Boston Air Quality

Exposure to pollutants such as NO_2, SO_2 and $PM_{2.5}$ are a significant concern, especially for those living in large cities. However, most major cities have five or fewer active air quality sensors. Various studies have shown that geostatistical models using traffic count, elevation, and land cover as variables can predict pollutant levels with high accuracy. However, collecting training data containing sufficient geospatial variation often involves large-scale deployment of sensors over the area of interest. In this study, we trained geospatial and spatio-temporal models for three EPA criteria pollutants – NO_2, SO_2, and $PM_{2.5}$ – using data collected from 398 counties across the US and applied the models to produce intra-urban pollution concentration levels for a 107.495 square mile region covering the Greater Boston area. The performance of our geospatial model (Land Use Regression) and spatio-temporal model (Gaussian Process) were found to be comparable of similar models in literature. Our study addresses also the public health challenge of effectively and meaningfully communicating scientific findings in environmental science to the general public. Specifically, we designed an interactive web interface for visualizing our Boston air pollution predictions. This interface serves as a proof-of-concept for an accessible, educational, and scientific tool.

Charlotte Orr, Drake University
A Comparative Assessment of ETD-RDP vs NDF for Simulating Calcium Release in Heart Muscle Cells

Calcium is integral to the function of the human heart. When an irregularity in the heart beat occurs, cardiac specialists need to be able to detect what caused the problem as quickly as possible. A mathematical model exists to model the calcium uptake by the heart. This 3D model is very complex, so does not have an exact solution. Therefore, numerical analysis is used to produce solutions. The current NDF solution is quite accurate, yet quite slow as it is based on an implicit scheme. Using the semi-implicit ETD-RDP scheme, we found that results can be produced in half the time, while only sacrificing one significant digit of accuracy in a 1D version of the aforementioned model. Currently, we are working to prove that the results will stay constant in a 2D model as well. It is our hope that we will be able to decrease the current 5 days it takes to produce results in order to improve the chances of cardiac specialists being able to detect malfunctions in the heart before they happen.

Kira Parker, University of Utah
Personalization of Indexed Content via Collaborative Filtering and Topic Modeling

The USC Shoah Foundation's Visual History archive hosts a large collection of videotaped interviews with genocide survivors and witnesses, each tagged with a number of keywords describing the content. Existing search tools for the archive rank relevant testimonies with respect to a query regardless of the information about the specific user who makes the query. We designed a collaborative latent semantic model to discover users' underlying preferences, which is turn are used to recommend new relevant testimonies and rank the query results in a personalized fashion. The core idea in ranking testimonies beyond naive relevance matching is to make connections between users' past queries and the testimonies through latent semantic spaces underlying the users' interest and the testimonies' semantic similarities surfaced via keywords. Specifically, we formulate the problem as completing a matrix that describes users' interests in different testimonies by joint factorization of matrices that describe the relevance of index terms to testimonies and user interests respectively. We test our proposed solution on USC Shoah Foundation's historical data and compare our results with the state of the art methods.

Jordan Pellett, University of Wisconsin-La Crosse
Modeling Chronic Vascular Responses Following a Major Arterial Occlusion

Peripheral arterial disease is a serious health concern characterized by a major arterial occlusion. Following an occlusion, blood supply to peripheral tissues is significantly reduced, leading to severe pain and reduced mobility. Here, mathematical modeling is used to investigate the role of different vascular segments in restoring blood flow following a major occlusion. Vascular adaptations to collateral arteries and the distal microcirculation have been observed to occur on both acute and chronic time scales. The model couples acute vascular responses to two chronic vascular responses–arteriogenesis (increased diameter of existing vessels) and angiogenesis (new vessel formation)–to predict the time frame and significance of vascular responses that help restore flow after an occlusion. Preliminary results suggest that more collaterals than distal vessels are required for optimal flow restoration. Ultimately, the model can be used to identify the most important vessels to target for future therapies.

Julie Phillis, Youngstown State University
see **Jacquelyn Chapman**

Lexi Rager, Youngstown State University
Helping Students Make Wise, Data-Driven Academic Decisions

As technology has advanced, humans have progressed to the point where they have billions of pieces of information at their fingertips. However, with that power often comes an overwhelming amount of data. To help people attend to only the most relevant items, information filtering systems sort through all known data and return only what is seen as significant to an individual. A specific type of information filtering system is called a recommender system, and it tries to unearth implicit and explicit information about a user, combine all new information with any previous data about that user, and finally use the whole body of knowledge to predict the preferences of the user as accurately as possible. Recommender systems have become commonplace in companies trying to give their customers a personalized experience. For example, Amazon recommends items to an individual that are consistent with their past shopping habits, generating many more sales than if it just recommending items at random. Our research takes recommender systems and applies them to higher education. After researching different types of recommender systems, studying their strengths and weaknesses, and compiling a system of our own, we have created a program that can help students predict what classes and professors they would most like taking at our university. It is our hope that by using our program, students will have a convenient resource tailored to them to help make wise, data-driven academic decisions.

Maddie Rainey, Grand Valley State University
Quantifying the Variability of Baseflow of Watersheds For the Chesapeake Bay

The U.S. Geological Survey National Water-Quality Assessment Project conducted a study of 225 sites in the Chesapeake Bay watershed to estimate base flow. Baseflow is the estimated volumetric discharge of water, primarily from groundwater sources, that is relayed to the measurement sites. The study is necessary in order to address the nation's water supply for changes in the environment. Baseflow is estimated using a recursive digital filter. Calculating the variability of baseflow water discharge is important to make informed decisions about water regulation. We explored the estimation of variability of baseflow using two methods: the bootstrap method and the Delta method. Each method has its own limitations and requirements. Ultimately, bootstrapping was shown to be a reasonable recommendation for estimating baseflow variability. The bootstrapping algorithm was parallelized in order to compute numerous iterations on multiple processors for big data analysis. The derivation of the variability of a non-constant streamflow was also considered for further study, but not implemented.

Tina Rajabi, Edmonds Community College
see **Joanne Beckford**

Catherine Rea, Benedictine College
Convergence of N^{th} Generalized Continued Fractions

This talk begins with a generalization of purely periodic simple continued fractions, in which we replace all numerators with a constant N and evaluate the limit of such a continued fraction as N approaches infinity. Next, we extend the generalization to eventually periodic continued fractions. We then look at simple continued fractions of generalized golden ratios divided by an integer. Finally, we prove results about when such expansions are one or two-periodic and discuss the exact form that the expansions take.

Mary Lib Saine, Furman University
Modeling Cilia-Driven Pulmonary Fluid Flow

Pulmonary cilia are flexible, densely packed, hair-like fibers that move with a coordinated whip-like motion to propel contaminants out of the lungs. The pulmonary cilia form a carpet on the surface of the lung and the bulk effect of the cilia's coordinated motion is a metachronal wave that travels along the carpet's surface. This wave helps propel the water-like periciliary liquid and the contaminants trapped in the adjacent mucus layer above it out of the lungs. We use the Method of Regularized Stokeslets to model the quasi-steady Stokes fluid flow generated by the immersed cilia. To model the cell surface that serves as a base for the cilia, we implement a system of regularized image singularities whose net effect creates a zero-velocity plane, replicating the cell surface. The dense spacing of pulmonary cilia creates modeling challenges. We investigate the fluid velocities in relation to the metachronal wave direction and the direction of the cilia's effective stroke to increase understanding of this biological transport mechanism.

Julie Sherman, University of Minnesota
Constraints on the Oceanic Carbon Sink Using Atmospheric Oxygen Data

Understanding the oxygen cycle and its relationship with the carbon cycle is an important tool in quantifying sources and sinks of carbon. Atmospheric measurements from the Scripps Oxygen Program have shown that while carbon dioxide levels are increasing, oxygen levels are decreasing, a trend consistent with their inverse relationship during photosynthesis and combustion. However, oxygen and carbon are decoupled with respect to oceanic exchanges, and thus this flux can be determined with the proper equations. In this study we develop a simple model of the global carbon-oxygen budget in which we incorporate data from the Scripps Carbon Dioxide and Oxygen Programs. Our results are obtained from derivative free optimization techniques, and give minimum sources and sinks necessary to replicate atmospheric observations. We compare our results to large and complex global circulation models.

Kasandra Short, Benedictine College
Reliability of Two-Terminal Graphs with Terminal Distance 3 or 4

The two-terminal reliability polynomial of a graph is the probability that a source vertex s is connected to a destination vertex t when each edge is included with probability p. A graph on a fixed number n vertices and m edges is uniformly most reliable if it has a two-terminal reliability polynomial greater than that of all other graphs with the same n and m for all $p \in [0, 1]$. We identify values of n and small m for which no uniformly most reliable graph exists when the distance from s to t is greater than or equal to 3. Additionally, the existence of a uniformly most reliable graph on specific values of n and large m is shown for graphs of distance 3.

Rebecca Sorsen, University of Nebraska-Lincoln
The Jones Polynomial of the Generalized Half Twist

The Jones Polynomial is a specific knot invariant than can yield extremely useful information; however, the Jones Polynomial is computationally complex. While computers can calculate the Jones Polynomial up to a certain complexity, we are interested in much larger, complex examples. Specifically, we focused on a special class of knots with many twisted, parallel strands, called the generalized half twist. We will discuss the properties of the Kauffman bracket expansion of the generalized half twist and apply our findings to the Jones Polynomial.

Lee Spence, Kennesaw State University
Physiologically Based Pharmacokinetic (PBPK) Modeling for a Persistent Chlorinated Water Contaminant

The compound 1,2,3-Trichloropropane (TCP) has historically been used during the production of pesticides and polysulfide rubbers, and remnant traces of TCP can still be found in groundwater. TCP shares structural similarities with toxic compounds, and it therefore is considered as possibly toxic. We developed a physiologically based pharmacokinetic (PBPK) model to determine internal doses of TCP in the body of F344 rats who were given an intravenous dose of TCP. Metabolism was modeled using Michaelis-Menton kinetics, and unknown metabolic parameters were optimized for using data from a 1984 intravenous study. Concentrations of TCP in each tissue with respect to time were predicted. We then altered the model to track internal doses in F344 rats that ingested TCP from contaminated drinking water, with drinking patterns modeled using a periodic intake function. Following our initial motivation, we scaled the drinking model to humans, with new parameters taken from literature or found through allometric scaling. We used this model to explore possible accumulation of TCP in human tissues after continued ingestion of contaminated water. Additionally, we performed sensitivity analysis on body fat percentage as it pertains to long-term accumulation.

Danika Van Niel, Syracuse University
Visualizing Dessin d'Enfants on the Torus

Suppose X is a compact, connected Riemann Surface. A Belyi map $\beta : X \to P^1$ is a rational function with at most three critical values. A Dessin d'Enfant is a bipartite graph obtained by considering the preimage of a path between two of these critical values. Such a graph is said to be planar if it can be embedded on the sphere such that its edges do not cross. Similarly, such a graph is said to be toroidal if it is not planar, yet can be embedded on the torus such that its edges do not cross. From the Dessin d'Enfant we are able to obtain a triple $(\sigma_0, \sigma_1, \sigma_\infty)$ of permutations in a symmetric group S_N, which satisfies $\sigma_0 \cdot \sigma_1 \cdot \sigma_\infty = 1$. The purpose of studying these concepts is to gain a deeper understanding of Belyi maps, and to create a database that provides quick information to researchers. The Dessin d'Enfants that will be explored are those that are regular toroidal graphs. There are patterns in the number of regular toroidal graphs depending on the number of faces in the Dessin d'Enfant. Regular toroidal graphs have special properties in graph theory that other Dessin d'Enfants don't necessarily have, and different types of regular toroidal graphs have unique properties as well. This work is part of PRiME with Chineze Christopher, Robert Dicks, Gina Ferolito, Joseph Sauder, and Danika Van Niel with assistance by Edray Goins and Abhishek Parab.

Katja Vassilev, Princeton University
Bifurcation Properties for Navigation Constellation Design

The Aerospace Corporation continually advises the Air Force on satellite improvement and maintenance for the Global Positioning System (GPS). The performance of GPS-like satellite constellations, referred to as Walker or Modified Walker Constellations, are of particular interest to the Aerospace Corporation. A Walker Constellation is a system of symmetrically spaced satellites designed for even coverage. They can be modified by altering the number and spacing of satellites and part of our research involves providing methods for modification of Walker Constellations. Our research involves analyzing how adjusting the semi-major axis and inclination of these GPS-like constellations affects the visibility and geometry of each constellation. A cost analysis determines the number of satellites that were viable at each semi-major axis. By varying the semi-major axes of these constellations, we find the bifurcation points corresponding to the number of satellites in a constellation required to achieve adequate visibility, geometry, and cost effectiveness.

Qimeng Yu, Carleton College
Automated jigsaw puzzle assembly and invariant signatures

"Turning and turning in the widening gyre The falcon cannot hear the falconer; Things fall apart; the centre cannot hold; Mere anarchy is loosed upon the world?" -William B. Yeats When things fall apart, who can put them back together? In this talk, we will present computational methods to reassemble blank jigsaw puzzles, relying on shape alone to determine assembly information. Using a measurement of shape known as an invariant signature, we discuss the process of finding matching puzzle pieces and algorithms for aggregating these matches into an assembled puzzle.