

# YVONNE LAI

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## Professional Experience

University of Nebraska-Lincoln. Assistant Professor, Department of Mathematics. 2013-current.

University of Michigan, Ann Arbor, MI. Assistant Research Scientist, School of Education, 2011-2013.  
(research specializations: mathematical knowledge for teaching, teacher education, practices of proving and reasoning).

University of Michigan, Ann Arbor, MI. NSF-RTG Assistant Professor (post-doctoral fellow) in Mathematics, 2008-2011. (research specializations: hyperbolic geometry, geometric group theory).

University of California, Davis, CA. 2002-2008. Graduate Research Student Assistant, Graduate Student Instructor, Graduate Teaching Assistant; Founder of the Davis Math Circle and co-founder of the Explore Math Program.

MIT, Cambridge, MA. 2000-2002. Recitation instructor for Department of Mathematics, mathematics instructor for the Experimental Studies Group.

**Areas of interest:** Mathematical Knowledge for Teaching, Teacher Education, Practices of Proving and Reasoning, Cognition of Proving and Reasoning and its Teaching

## Education

University of California, Davis, CA. Ph.D. 2008. (advised by Michael Kapovich).

MIT, Cambridge, MA. S.B. Mathematics, 2002.

## Honors

- 2012 *Janet Duffin Award* from the British Society for Research into Learning Mathematics (BSRLM) for most outstanding contribution to journal *Research in Mathematics Education*.
- 2010 *Best Paper Award* from the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA-RUME).

## Grants

- NSF EAGER Grant (**awarded** for 2014-2016). Howell, H. (PI), Lai, Y., & Phelps, G. (co-PIs). *Building Understanding and Measurement of Secondary Mathematical Knowledge for Teaching (MKT)* (Joint awards #DGE-1445551 administered through UNL, \$75,550.00, and #DGE-1445630 administered through Educational Testing Service, \$204,356.00)
- NSF Noyce Grant (**awarded** for 2014-2017). Smith, W. (PI), Lai, Y., Lewis, W.J., & Males, L. (co-PIs). *NebraskaNOYCE Phase II: Investigating the Impact in High-Need Districts* (#DUE-1439867, \$299,878.00)
- NSF Noyce Grant (**awarded** for 2010-2016). Lewis, W.J. (PI), Swidler, S., Smith, W., Lai, Y., Males, L. (co-PIs), Papick, I., Fowler, D., & Kauffman, D. (former co-PIs). *NebraskaNOYCE: NSF Mathematics Teaching and Master Teaching Fellows Program* (#DUE-1035268, \$3,000,000.00)
- Research Development Program Fellow (**awarded** for 2013-2014). University of Nebraska-Lincoln. One of 15 young faculty selected for professional development program.
- University of Michigan CRLT-ISL Grant (**awarded** for 2009-2010). Awarded to ~30 faculty members university-wide. Funded project to study mathematicians' writing and revision of proof.
- NCTM/NSF International Travel Grant for ICME-11 (**awarded** for 2008). Awarded to ~30 mathematicians, educators, and graduate students nationwide for travel to the 11th International Congress on Mathematical Education in Monterrey, Mexico.

## Professional Society Memberships

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**Professional Society Memberships:** American Educational Research Association (AERA), Association of Mathematics Teacher Educators (AMTE), American Mathematical Society (AMS), Mathematical Association of America (MAA)

## Professional Writing (Education)

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- Lai, Y. & Weber, K. (2014). Factors mathematicians consider when creating pedagogical proof. *Educational Studies in Mathematics*, 85(1), 93-108. (DOI: 10.1007/s10649-013-9497-z)
- Lai, Y., Weber, K., & Mejía-Ramos, J.-P. (2012). Mathematicians' perspectives on features of a good pedagogical proof. *Cognition & Instruction* 30(2), 146-169. (DOI: 10.1080/07370008.2012.661814)  
(This paper reports on the study discussed in the CRUME 2010 proceedings and a follow-up study.)
- Samkoff, A., Lai, Y., & Weber, K. (2012). How mathematicians use diagrams to construct proofs. *Research in Mathematics Education* 14(1), 49-67. (DOI: 10.1080/14794802.2012.657438) (Winner of 2012 *Janet Duffin Award* for most outstanding contribution to journal)
- Lai, Y. (2012). *Teaching Undergraduate Mathematics*. Critical Issues in Mathematics Education Series, Volume 5. Mathematical Sciences Research Institute, Berkeley, CA. (I am the author of this manuscript that synthesizes the 2009 Critical Issues in Mathematics Education held at MSRI, on the theme of undergraduate mathematics teaching; this manuscript is available at <http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm460>.)
- Lai, Y. & Weber, K. (2010). What makes good proof: Investigating mathematicians' proof revisions. *Proceedings of the 13th Conference on Research in Undergraduate Mathematics Education (CRUME)*. (Winner of the 2010 *Best Paper Award, SIGMAA on Research in Undergraduate Mathematics Education*.)
- Lai, Y., McCallum, W., Soto-Johnson, H. (2008). *The role of universities in preparing teachers*. Technical report on ICME-11 for NCTM/NSF committee.

## Professional Writing (Mathematics)

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- **Dissertation.** Lai, Y. (2010). An effective compactness theorem for Coxeter groups. *Geometriae Dedicata* 145(1), 195-217. (DOI: 10.1007/s10711-009-9416-8. arXiv:0902.2718.) *Key words:* Coxeter groups, hyperbolic geometry, geometric group theory.
- **Lessons in Geometry.** Proofread and contributed solutions to problems from Hadamard's *Leçons de géométrie élémentaire*, Vol. I, Ch. V, 13th edition, on inversive geometry, for companion volume by Mark Saul.

## Presentations (Education)

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- *Secondary Mathematical Knowledge for Teaching and its Relationship to Elementary Mathematical Knowledge for Teaching*. Annual meeting of the American Educational Research Association (AERA 2015). Chicago, IL. March 2015. With Heather Howell and Geoffrey Phelps (Educational Testing Service).  
\*\*\* For AERA, researchers write a paper to present and submit this paper as a part of a proposal to speak. Proposals are reviewed; not all proposals are accepted. The AERA is the largest professional community of US educational researchers.
- *Articulating Structure and Regularity in Rich Mathematical Tasks*. Annual meeting of the Association of Mathematics Teacher Educators (AMTE 2015). Orlando, FL. February 2015. With Dave Kennedy (Shippensburg University), Diana Sherman (University of Michigan), Judith E. Jacobs (JEJ Math, Ltd.).  
\*\*\* For AMTE, mathematics teacher educators submit a proposal to speak. Proposals are reviewed; not all proposals are accepted.

- *From here, to there, to everywhere: Connecting mathematical time scales to improve teacher education* Special Session of the American Mathematical Society (AMS) on the theme Creating Coherence in K-12 Mathematics. Joint Mathematics Meeting (JMM 2015). San Antonio, TX. January 11, 2015. With Mary Alice Carlson (Montana State University) and Ruth Heaton (University of Nebraska-Lincoln).  
\*\*\* Presentations at special sessions of the JMM are given by invitation only. The American Mathematical Society is the largest professional society for US research mathematicians.
- *Building understanding and measuring secondary mathematical knowledge for teaching.* Poster presented at the convening of NSF-EAGER principal investigators. Arlington, VA. October 26, 2014. With Heather Howell and Geoffrey Phelps (Educational Testing Service).
- *The role of definitions in mathematical literacy.* Invited presentation to undergraduate seminar course on mathematical literacy at the University of Michigan. September 16, 2014.
- *Clear instruction of mathematical practice: In defense of rich and ordinary problems and how we can use them to teach teaching.* Presentation for the Center for Mathematics and Science Education of the University of Utah. Salt Lake City, UT. April 22, 2014.
- *Tasks assessing mathematical knowledge for teaching as representations of mathematics teaching practice.* Annual meeting of the American Educational Research Association (AERA 2014). Philadelphia, PA. April 5, 2014. With Heather Howell (Educational Testing Service).
- *Mathematicians engaged in mathematics education.* Invited plenary presentation at the Critical Issues in Mathematical Education at the Mathematical Sciences Research Institute (CIME 2014 at MSRI). Berkeley, CA. March 28, 2014. With James Epperson (University of Texas at Arlington), Dev Sinha (University of Oregon), and Sunita Vatuk (City College of New York).
- *Introduction to Mathematical Knowledge for Teaching.* Mathematicians in Mathematics Education (MIME 2014). Collaboration between the Institute for Mathematics & Education (IME), Tucson, AZ, and the Texas A&M College of Education and Human Development. College Station, TX. March 20, 2014. 3-hour plenary workshop presentation.
- *Tasks assessing mathematical knowledge for teaching as representations of mathematics teaching practice.* Conference on Research on Undergraduate Mathematics Education (RUME 2014). Denver, CO. March 1, 2014. With Heather Howell (Educational Testing Service).  
\*\*\* For RUME, researchers write a paper to present and submit this paper as a part of a proposal to speak. Proposals are reviewed; not all proposals are accepted.
- *Teaching teachers to recognize and use the mathematical practices.* Annual meeting of the Association of Mathematics Teacher Educators (AMTE 2014). Irvine, CA. February 7, 2014. With Dave Kennedy (Shippensburg University), Diana Sherman (University of Michigan), Judith E. Jacobs (JEJ Math, Ltd.).
- *Finding footholds: Charting a route to mathematics education research.* Invited presentation to the international workshop Mathematicians and School Mathematicians Education – a Pan-American workshop at the Banff International Research Station. Banff, Alberta, Canada. January 30, 2014.
- *Clear instruction of mathematical practice: Preparing teachers to use rich and ordinary problems to teach the Common Core Standards for Mathematical Practice.* Special Session of the American Mathematical Society (AMS) on the theme of Changing Education of Pre-Service Teachers at the Joint Mathematics Meeting (JMM 2015). Baltimore, MD. January 15, 2014.
- *Setting up and enacting approximations of mathematical tasks of teaching in a practice-focused curriculum.* Annual meeting of North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA 2013). Chicago, IL. November 16, 2013. With Yeon Kim.  
\*\*\* For PME-NA, researchers write a paper to present and submit this paper as a part of a proposal to speak. Proposals are reviewed; not all proposals are accepted.

- *Rotations and reflections, what they have to do with each other, algebra, and the Common Core*. Invited presentation to the Lincoln Mathematics Teacher Circle. Lincoln, NE. November 14, 2013.
- *The role of definitions in mathematical literacy with a focus on geometric transformations*. Invited presentation to undergraduate seminar course on mathematical literacy at the University of Michigan. Ann Arbor, MI. September 17, 2013.
- *The role of pedagogical context in items assessing mathematical knowledge for teaching*. Annual meeting of the American Educational Research Association (AERA 2013). Chicago, IL. April 29, 2013. With Erik Jacobson and Mark Thames.
- *Polynomials, lemmas, and a fundamental theorem: A partial proof of concept of teaching the concept of proof*. Loyola Mathematics Teaching Seminar. Chicago, IL. April 15, 2013.
- *Introduction to Mathematical Knowledge for Teaching*. Mathematicians in Mathematics Education (MIME 2012). Institute for Mathematics & Education (IME), Tucson, AZ. March 26, 2013. 3-hour plenary workshop presentation.
- *Some mathematicians' perspectives on pre-service mathematics courses*. Conference on Research in Undergraduate Mathematics Education (RUME 2013). Denver, CO. February 21, 2013. With Elham Kazemi (University of Washington).
- *Using practices to organize instruction of mathematics for prospective teachers*. Conference on Research in Undergraduate Mathematics Education (RUME 2013). Denver, CO. February 20, 2013.
- *Improving learning and teaching by investigating mathematical knowledge for teaching*. University of Nebraska-Lincoln, Mathematics Colloquium. February 6, 2013.
- *Improving learning and teaching by investigating mathematical knowledge for teaching*. University of Arizona, Mathematics Colloquium. Tucson, AZ. January 17, 2013.
- *Learning to do mathematics as a teacher*. University of Victoria Mathematics Colloquium. Victoria, British Columbia, Canada. October 13, 2012.
- *Representing teaching in measure of mathematical knowledge for teaching: The role of pedagogical context*. Representations of Mathematics Teaching workshop at the University of Michigan. Ann Arbor, MI. June 7, 2012. Joint presentation with Mark Thames reporting on research work joint with Erik Jacobson, Kahye Kim, Yeon Kim, and Rohen Shah.
- *Preparing teachers and teacher educators in mathematics departments: Mathematical Knowledge for Teaching*. Mathematicians in Mathematics Education (MIME 2012). Institute for Mathematics & Education (IME), Tucson, AZ. April 14, 2012. 3-hour plenary workshop presentation.
- *Features of instructional support and collective work in teaching prospective elementary teachers*. Joint Mathematics Meetings (JMM 2012). Boston, MA. January 6, 2012. Joint presentation with Wendy Aaron and Hyman Bass reporting on work with Deborah Ball, Laurie Sleep, Kara Suzuka, and Mark Thames.
- *Supporting mathematicians teaching teachers*. University of New Hampshire Mathematics Colloquium. Durham, NH. Dec 9, 2011.
- *More than the problem list: Prospective practices for prospective teachers*. University of Michigan Math/Teaching Seminar. Ann Arbor, MI. October 24, 2011. With Hanna Bennett and Mark Radosevich (University of Michigan).
- *Features of mathematical tasks that can support the learning and teaching of mathematical practices*. University of Michigan Math/Teaching Seminar. Ann Arbor, MI. Nov 21, 2011. Joint with Hyman Bass (University of Michigan).

- *Preparing future teachers*. Invited presentation to the Inquiry-Based Learning (IBL) Workshop 2011. Ann Arbor, MI. May 23, 2011. With Hanna Bennett, Moon Duchin, Mark Radosevich, and Alejandro Uribe (University of Michigan).
- *Learning to do mathematics as a teacher*. University of Oregon Mathematics Colloquium. Eugene, OR. May 2, 2011.
- *Proof and proving in a secondary content course*. Oklahoma State University Education Seminar. Stillwater, OK. April 29, 2011.
- *Learning to do Mathematics as a teacher*. Mathematicians in Mathematics Education (MIME 2012). Institute for Mathematics & Education (IME), Tucson, AZ. April 23, 2011. 3-hour plenary workshop presentation.
- *What is mathematical problem solving for teaching?*. Workshop on MKT at the Secondary Level. Institute for Mathematics & Education (IM&E), Tucson, AZ. March 25, 2011.
- *Improving Proofs for Pedagogical Purposes: Two Studies*. Conference on Research on Undergraduate Mathematics Education. Portland, OR. February 25, 2011. With Keith Weber and Juan-Pablo Mejía-Ramos (Rutgers University).
- *Designing materials to support the learning of K-6 mathematics teacher educators*. Association of Mathematics Teacher Educators (AMTE 2011). Irvine, CA. January 28, 2011. With Suzanne Chapin, Ziv Feldman, Matthew Chedister, Diana Cheng (Boston University), Deborah Ball, Hyman Bass, Diana Sherman, Laurie Sleep, Kara Suzuka, and Mark Thames (University of Michigan).
- *San Francisco Math Circle (SFMC) Mathematics Attitudes and Community Survey Development and Evaluation Tool*. Mathematical Association of America (MAA) Session on Fostering, Supporting, and Propagating Math Circles for Students and Teachers at the Joint Mathematics Meetings (JMM 2011). New Orleans, LA. January 8, 2011. With Brandy Wieggers (San Francisco State University).
- *How mathematicians use diagrams in proving*. Special Interest Group on Research on Undergraduate Mathematics Education (SIGMAA-RUME) Session at the Joint Mathematics Meeting (JMM 2011). New Orleans, LA. January 7, 2011. With Aron Samkoff and Keith Weber (Rutgers University).
- *The role of definitions in teaching, learning, and doing mathematics: An example of MKT*. DePaul University Mathematics Teaching Seminar. Chicago, IL. May 28, 2010. With Mark Thames, Deborah Ball, Hyman Bass, and Laurie Sleep.
- *Proof and proving in a secondary content course*. University of Michigan Math/Teaching Seminar. Ann Arbor, MI. April 5, 2010.
- *Between construction and communication: What happens during proof revision?* Conference on Research in Undergraduate Mathematics Education (RUME 2010). Raleigh, NC. February 27, 2010. With Keith Weber (Rutgers University).
- *Between construction and communication: What happens during proof revision?* University of Arizona Mathematics Colloquium. Tucson, AZ. January 22, 2010. With Keith Weber (Rutgers University).
- *Michigan Algebra Project report: Implementation and research*. Algebra Project Cohorts Research Planning Meeting. Miramar, Florida. January 31, 2010. With Laura Roop and Mark Thames (University of Michigan).
- *Variations on a theme: Mathematics graduate student instructor training*. Critical Issues in Mathematics Education at the Mathematics Sciences Research Institute (CIME 2009 at MSRI). Berkeley, CA. May 12, 2009.
- *Davis Math Circle: Lessons from four years (and counting!) of vertical integration*. MSRI Great Circles Workshop 2009. Berkeley, CA. April 22, 2009.
- *Report on ICME-11: The role of universities in preparing teachers* Annual meeting of the National Council of Teachers of Mathematics (NCTM 2009). Washington, D.C. April 19, 2009. With William McCallum (University of Arizona) and Steven Williams (Lock Haven University of Pennsylvania).

- *Teaching math majors how to teach.* Conference on Research on Undergraduate Mathematics Education (RUME 2009). Portland, OR. March 1, 2009. With Marion Moore and Hillel Raz (University of California-Davis).
- *The role of competitions in the high school curriculum.* Invited panel speaker to represent past participants of the USAMTS, a monthly competition in which high school students submit carefully written proofs for a set of problems. Moderator: Mark Saul. Annual conference of the National Council of Teachers of Mathematics (NCTM 2000). Chicago, IL. April 4, 2000.

### **Other Professional Meetings Attended**

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- Nebraska Math & Science Summit. Lincoln, NE. December 2014.
- Conference of the Nebraska Association of Teachers of Mathematics (NATM). Kearney, NE. September 2014.
- Noyce Midwest Regional Conference. Omaha, NE. September 2014.
- Exploring the Possibility for a Common Core Mathematics Licensure Examination for Entry into Teaching. University of Michigan, School of Education, Ann Arbor, MI. May 2013. Authored a meeting brief narrating the key discussion points and arguments by presenters and participants, sent to all participants after the meeting.
- Critical Issues in Mathematics Education (CIME). Mathematical Sciences Research Institute (MSRI), Berkeley, CA. March 2013.
- 12th International Congress on Mathematics Education (ICME-12). Seoul, Korea. July 2012.
- Conference Board of the Mathematical Sciences (CBMS). Washington, D.C. October 2011.
- 11th International Congress on Mathematics Education (ICME-11). Monterrey, Mexico. July 2008.

### **Selected Presentations (Mathematics)**

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- *Decompositions of polyhedra: From scissors congruence to Banach-Tarski.*
  - Canada/USA Mathcamp Opening Colloquium. Portland, OR. (2014)
  - Invited plenary at the Undergraduate Mathematics Symposium 2011, University of Illinois-Chicago. (2011)
- *An effective compactness theorem for Coxeter groups.*
  - AMS Sectional Meeting held at Wesleyan, CT (2008)
  - Yale University (2008)
  - Geometric Groups on the Gulf (2008)
  - The Ohio State University (2008)
  - Michigan State University (2008)
  - University of Michigan (2008)
  - Smith College (2007), Tufts University (2007)
  - Boston College (2007)
  - Miami University (2007)
  - Binghamton University (2007)
  - Cornell University (2007).

## Selected Professional Activities and Service

- ◊ **Undergraduate Mentor**, 2014-2015. Department of Mathematics, University of Nebraska-Lincoln. Met with an undergraduate students.
- ◊ **Teaching Mentor**, 2013-2015. Department of Mathematics, University of Nebraska-Lincoln. Observed and provided feedback to 3 graduate students on their teaching, provided guidance on 4 graduate student and 1 post-doctoral fellow's teaching statement.
- ◊ **Teaching Advisory Committee**, 2013-2014, 2014-2015. Department of Mathematics, University of Nebraska-Lincoln. Committee member.
- ◊ **National Science Foundation**, 2014, 2010. Reviewer.
- ◊ **Smarter Balanced Assessment Consortium**, 2013-2014. Reviewed mathematics assessments items for K-12 students for alignment to Common Core State Standards.
- ◊ **National Association of Math Circles**, 2012-2015. Advisory Board Member. Mini-Grants Subcommittee member.
- ◊ **RUME**, 2011, 2013, 2014, 2015. Reviewer for conference presentation proposals to be given at the 2011 Conference on Research for Undergraduate Mathematics Education.
- ◊ **MIME Workshop (Mathematicians in Mathematics Education)**, 2011-2014. Co-Organizer with William McCallum, Roger Howe, and Yeping Li (2014). Co-Organizer with William McCallum (2013). Co-Moderator with William McCallum and Roger Howe (2012); with William McCallum (2011). Institute for Mathematics & Education, University of Arizona, Tucson, AZ.
- ◊ **Canada/USA Mathcamp**, 2000-2012. Academic Coordinator (2011), Academic Co-Coordinator (2008), Instructor Hiring Committee (various years), Counselor Hiring Committee (various years).
- ◊ **AWM (Association for Women in Mathematics)/MfA (Math for America) Essay Contest**, 2012. Round 2 Judge for high school essays describing interviews with living female mathematicians.
- ◊ **NCTM Research Preession**, 2011. Discussant for paper on problem-solving beliefs and paper on teaching and learning infinite series.
- ◊ **Elementary Mathematics Laboratory**, 2010. Assisted design and facilitation of workshop on Mathematical Knowledge for Teaching Teachers, led by Hyman Bass.
- ◊ **Algebra Project**, 2007, 2009-2010. Lesson planning team member for Ypsilanti High School Algebra Project class (2009-2010); implementation part of NSF DRK12 grant (#DRL-0822715). Curriculum Development Meeting (2007); one of three invited Young Mathematicians.
- ◊ **Bay Area Mathematics Olympiad for Teachers**, 2010. "Chief inspirer" to director Joshua Zucker. Consulted about olympiad design; contributed and graded problems.
- ◊ **National Research Council Committee to Evaluate the NSF-VIGRE Program**, 2007. Represented the UC Davis VIGRE program.
- ◊ **IAS Program for Women and Mathematics**, 2008. Teaching Assistant for course on hyperbolic geometry and orbifolds, taught by Genevieve Walsh.
- ◊ **Explore Math Program/Davis Math Circle**, 2005-2008. Co-founder. Local proposal for program incorporated as Section 3.3 of successful departmental 5-year NSF VIGRE proposal (#DMS-0636297). Co-authored section with B. Wieggers, S. Williams, and S. Michalakis.
- ◊ **Public Service Center IAP Fellow**, 1998. Assisted physics and chemistry classes at Cambridge Rindge and Latin High School in Cambridge, MA, for January term.

## Research advising

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- Hayley Andersen (undergraduate)
- Erica Gilliland (graduate)

## Teaching

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**Mathematics courses taught or to be taught at UNL:** Capstone Courses I and II for Secondary Mathematics Teaching (Math 404/807, Math 408/808), Discrete Mathematics for Secondary Mathematics Teaching (Math 405).

**Mathematics courses taught outside of UNL:** Content for Secondary Mathematics Teaching (U Michigan), Ordinary Differential Equations (UC Davis, MIT), Precalculus (U Michigan).

Content for Secondary Mathematics Teaching focused on mathematics bridging high school mathematics with elective coursework, such as group theory and real analysis, as well as elementary and middle school mathematics. Homework and in-class discussions connected topics to pedagogical content knowledge and mathematical knowledge for teaching as characterized Shulman (1986) and Ball, Thames, and Phelps (2008). (Articles at <http://dx.doi.org/10.2307/1175860> and <http://dx.doi.org/10.1177/0022487108324554>).

**Mathematics courses assisted:** Riemann Surfaces (graduate level; UC Davis), Differential Geometry (UC Davis), History of Mathematics (UC Davis), Ordinary Differential Equations (UC Davis, MIT), Vector Calculus (UC Davis, MIT), Calculus (UC Davis, MIT).

**Mathematics courses coordinated:** Precalculus (U Michigan). Organized and led weekly meetings for instructors, with focus on student thinking and pedagogy for teaching concepts such as inverse functions and transformations of functions. Designed common exams and coordinated common grading. Approximately 600 student enrollment, 24 sections.

**Education courses taught:** Methods for Secondary School Mathematics Teaching (U Michigan).

Focused on teaching practices addressing the dimension of “richness of mathematics” in Mathematical Quality of Instruction, an observation protocol whose scores correlate with mathematical knowledge for teaching (Hill et al., 2008) and student outcomes (Kane & Staiger, 2012). (Articles at <http://dx.doi.org/10.1080/07370000802177235> and <http://www.metproject.org/reports.php>).

**Special program teaching/mentoring:** Canada/USA Mathcamp 2000-present, Bay Area Teacher’s Circle 2009, San Francisco Math Circle 2009, Oakland/East Bay Math Circle 2007-2009, Davis Math Circle 2006-2008, UC Davis Research Experience for Undergraduates 2005, California Summer School for Math and Science (COSMOS) 2003.

Canada/USA Mathcamp classes include:

- *Slicing and Dicing* (5 hours lecture) – Scissors Congruence Problem, the Dehn invariant for polyhedra, and the Banach-Tarski Paradox. Based on notes by Danny Calegari on Scissors Congruence and the Dehn invariant.
- $SL(2, \mathbb{Z})$  (7 hours lecture) – from the perspectives of geometric group theory, combinatorial group theory, and hyperbolic geometry.
- *The Kakeya Problem* (1 hour lecture) – based on A. S. Besicovitch, The Kakeya Problem, *Am. Math. Monthly*, Vol. 70(7): 697 – 706 (1963).
- *Musical Orbifolds* (5 hours lecture/discussion) – based on Callender, C., Quinn, I., & Tymoczko, D., Generalized Voice-Leading Spaces, *Science* 320: 346-348 (2008), and Tymoczko, D., The Geometry of Musical Chords, *Science* 313: 72-74 (2006).
- *The Dwarf Problem and Heat Flow* (10 hours lecture) – an introduction to complex vector spaces, eigenvectors and eigenvalues, and discrete Fourier transforms.