

Department of Mathematics

Graduate Advisory Committee
February 2012

Discrete Mathematics Qualifying Exam Syllabus

This material is covered in the introductory graduate sequence Math 850 and Math 852.

1. Enumeration

- (1) Basic Counting Principles. Counting permutations and combinations. Binomial coefficients and multinomial coefficients. Combinatorial identities. Partitions. Stirling numbers of the first and second kind. Catalan numbers. The Twelffold Way.
- (2) Recurrence relations. Solving recurrences via the characteristic equation method and generating functions.
- (3) Ordinary and exponential generating functions. Exponential Formula.
- (4) Principle of Inclusion-Exclusion.
- (5) Burnside's Lemma.
- (6) The basic probabilistic method and linearity of expectation.

2. Discrete Structures: Coding Theory, Design Theory, Posets

- (1) Basic concepts of Coding Theory. Hamming metric. Parameters. Shannon's Theorem.
- (2) Linear codes. Properties of generator matrix and parity check matrix. Syndrome decoding. Hamming codes.
- (3) Sphere packing, Singleton, and Gilbert-Varshamov bounds. Asymptotic bounds.
- (4) Generalized Reed-Solomon codes.
- (5) Basic concepts of posets. Dilworth's Theorem. Sperner's Theorem. LYM Inequality. Erdős-Ko-Rado Theorem.
- (6) Basic concepts of designs. Relation of parameters. Fisher's Inequality. Nonexistence results. Finite projective planes.

3. Graph Theory

- (1) Basic concepts of graphs. Graphic sequences. Havel-Hakimi Theorem. Eulerian graphs. Characterization of trees. Minimum weight spanning trees and algorithms. Matrix Tree Theorem.
- (2) Matchings in bipartite and non-bipartite graphs. Hall's Theorem. König-Egeváry Theorem. Tutte's 1-factor Theorem.
- (3) Connectivity and edge-connectivity. Menger's Theorem. Network flows. Ford-Fulkerson algorithm. Max Flow-Min Cut Theorem.
- (4) Vertex and edge coloring. Greedy bounds. Szekeres-Wilf Theorem. Brooks' Theorem. Turán's Theorem. Vizing's Theorem and Shannon's Theorem.
- (5) Planar graphs. Outerplanar graphs. Euler's Formula. Kuratowski's Theorem. Five Color Theorem. Statement of Four Color Theorem.
- (6) Hamiltonian cycles. Dirac's Theorem. Ore's Theorem.

References

- Douglas B. West, *Combinatorial Mathematics*, prepublication version, 2011.
- Douglas B. West, *Introduction to Graph Theory*, 2nd edition, Prentice Hall, 2001.
- Peter J. Cameron, *Combinatorics: Topics, Techniques, Algorithms*, Cambridge University Press, 1995.
- Béla Bollobás, *Modern Graph Theory*, Springer, 1998.
- Herbert S. Wilf, *Generatingfunctionology*, 3rd edition, A. K. Peters, 2006.
- Ron M. Roth, *Introduction to Coding Theory*, Cambridge University Press, 2006.
- Vera Pless and W. Cary Huffman, *Fundamentals of Error-Correcting Codes*, Cambridge University Press, 2003.