Quiz 4 **Math 208** **Fall 2009**

Name:__________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14), and you should make obvious simplifications. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is optional.

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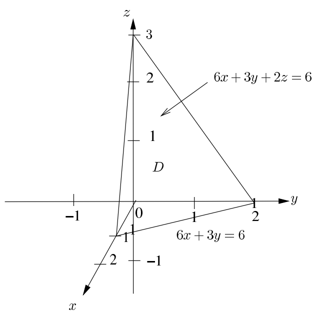
(6) **1.** (Exercise 13.5.3) Write two different iterated integrals for the volume of the tetrahedron cut off in the first octant by the plane $6x + 3y + 2z = 6$. Sketch the solid.

SOLUTION.

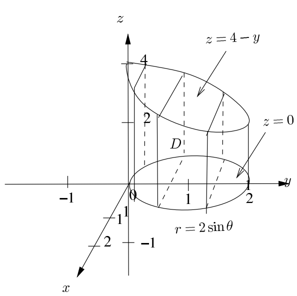


From the graph we see that

$$\iiint_D 1 \cdot dV = \int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} 1\, dz\, dy\, dx = \int_0^2 \int_0^{1-\frac{1}{2}y} \int_0^{3-3x-\frac{3}{2}y} 1\, dz\, dx\, dy = \int_0^3 \int_0^{2-\frac{2}{3}z} \int_0^{1-\frac{1}{2}y-\frac{1}{3}z} 1\, dx\, dy\, dz.$$

(6) **2.** (Exercise 13.7.15) D is the solid inside the right circular cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and top is given by $z = 4 - y$. Set up an iterated integral for $\iiint_D f(x,y,z)\, dV$ in cylindrical coordinates in the order $dz\, r\, dr\, d\theta$. Sketch D.

SOLUTION.

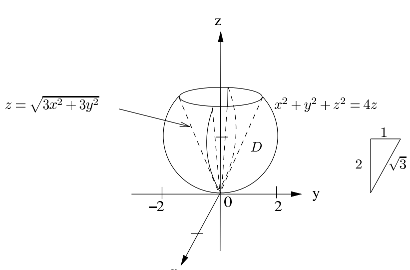


Cylinder $r = 2\sin\theta$ is the same as $r^2 = 2r\sin\theta$, that is, $x^2 + y^2 = 2y$, $x^2 + (y-1)^2 = 1$, a circle of radius 1, center at $(0,1)$. The bottom of the solid is $z = 0$ and the top is the plane $z = 4 - y$. Thus $\iiint f(x,y,z)\, dV$ is the iterated integral

$$\int_0^{\pi} \int_0^{2\sin\theta} \int_0^{4-r\sin\theta} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta.$$

(8) **3.** (Exercise 5, Handout) Express the mass of an object inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$ as an iterated integral in spherical coordinates, if the density is given by $\delta(x,y,z) = \frac{1}{x^2+y^2+z^2}$. Sketch the solid.

SOLUTION.



Complete the square to see the sphere is given by $x^2 + y^2 + (z-2)^2 = 4$, so has radius 2, center at $(0,0,2)$. In spherical coordinates it is $\rho^2 = 4\rho\cos\phi$ or $\rho = 4\cos\phi$. Cone angle from the vertical is the same as the line $z = \sqrt{3}y$ with the z-axis. Take $y = 1$, get $z = \sqrt{3}$, 30-60-90 right triangle as in the graph, so angle ϕ starts at $\phi = \pi/6$. Hence the mass is $M =$

$$\iiint_D \delta(x,y,z)\, dV = \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{4\cos\phi} \frac{1}{\rho^2}\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{4\cos\phi} \sin\phi\, d\rho\, d\phi\, d\theta$$