Quiz 3 **Math 208** **Fall 2009**

Name:______________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14), and you should make obvious simplifications. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is optional.

---

(8) **1.** (Exer. 1, 2nd Deriv. Handout) Find and classify (as local maxima, local minima or saddle points) the critical points of $f(x,y) = 8x^2 + 4x^2y + y^2 - 7$. (Check: $(x,y) = (0,0), (\pm 1, -2)$.)
SOLUTION. First calculate

$$\begin{aligned} f_x &= 16x + 8xy \\ f_{xx} &= 16 + 8y \\ f_{xy} &= 8x \\ f_y &= 4x^2 + 2y \\ f_{yy} &= 2 \\ D_f &= \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = (16+8y)\,2 - (8x)^2 = 32 + 16y - 64x^2. \end{aligned}$$

Next find critical points via $0 = f_x = 8x(2+y)$, $0 = f_y = 4x^2 + 2y$, so that $x = 0$ or $y = -2$. In either case $y = -2x^2$, so that $(0,0)$ is critical, as is $(\pm 1, -2)$.
Finally, check discriminant: $D_f(0,0) = 32 > 0$, so we have local max/min. Since $f_{yy} = 2 > 0$, f has a local min at $(0,0)$. At $(\pm 1, -2)$ we have $D_f(\pm 1, -2) = 32 - 32 - 64 < 0$, so f has saddle points at $(\pm 1, -2)$.
(8) **2.** (Exer. 12.8.37) Find the extreme values of $f(x,y) = x^2y + 1$ on the circle $x^2 + y^2 = 9$. (Check: $(x,y) = (0, \pm 3)$, $\left(\pm\sqrt{6}, \pm\sqrt{3}\right)$.)
SOLUTION. Set $g(x,y) = x^2 + y^2 - 9$ and find the critical points satisfying $\nabla f(x,y) = \lambda \nabla g(x,y)$, and $g(x,y) = 0$, that is,

$$\begin{aligned} 2xy &= 2\lambda x \\ x^2 &= 2\lambda y \\ x^2 + y^2 &= 9. \end{aligned}$$

First equation factors into $x(y - \lambda) = 0$, so $x = 0$ or $y = \lambda$. If $x = 0$, then $y^2 = 9$ from the third equation, so $y = \pm 3$ and $\lambda = 0$. If $y = \lambda$, then $x^2 = 2y^2$ from the second equation and so $2y^2 + y^2 = 9$ from the third, and hence $y = \pm\sqrt{3}$. Thus $x^2 = 2y^2 = 6$ and $x = \pm\sqrt{6}$. Now check the value of f at each critical point and obtain

$$\begin{aligned} f(0, \pm 3) &= 0^2 \cdot (\pm 3) + 1 = 1 \\ f\left(\pm\sqrt{6}, \sqrt{3}\right) &= \left(\pm\sqrt{6}\right)^2 \sqrt{3} + 1 = 6\sqrt{3} + 1 \\ f\left(\pm\sqrt{6}, -\sqrt{3}\right) &= -\left(\pm\sqrt{6}\right)^2 \sqrt{3} + 1 = -6\sqrt{3} + 1. \end{aligned}$$

So f has maximum value $6\sqrt{3} + 1$ at $\left(\pm\sqrt{6}, \sqrt{3}\right)$, minimum value $-6\sqrt{3} + 1$ at $\left(\pm\sqrt{6}, -\sqrt{3}\right)$.
(4) **3.** (Exam 1) Let $f(x,y) = \sqrt{y^2 - x^2}$. Compute the total differential of this function.
SOLUTION.

$$df = f_x dx + f_y dy = \frac{-2x\,dx}{2\sqrt{y^2 - x^2}} + \frac{2y\,dy}{2\sqrt{y^2 - x^2}} = \frac{-x\,dx + y\,dy}{\sqrt{y^2 - x^2}}.$$