Math 208
Chain rule additional problems

In these problems, write down the appropriate version of the multivariable chain rule and use it to find the requested derivative.

1. Find w_t in terms of x, y, s and t if $x(s,t) = s\cos(2t)$, $y(s,t) = 2t - s$, and $w(s,t) = f(x(s,t), y(s,t))$ with $f(x,y) = x\tan^{-1}(y)$.

2. Find $\frac{\partial w}{\partial t}$ in terms of x, y, z, r, s and t, if $w = xy^2e^{3z}$, $x = r^2t^3$, $y = 4s - 6t$, and $z = r^3 + 5st$.

3. Find $\frac{dt}{ds}$ in terms of x, y, z, w and s if $t = xe^{xy} + w^3yz$, $x = s^2$, $y = 5s - 1$, $z = \cos(s)$, and $w = \frac{2}{s}$.

4. If $f(x,y)$ satisfies f_x and f_y are continuous everywhere, $f_x(-6,2) = 5$, $f_y(-6,2) = -3$, and $g(s,t) = f(3st, \frac{8s}{t^2})$, find $g_t(1,-2)$.

5. Find $\left.\frac{\partial w}{\partial t}\right|_{(s,t)=(-1,2)}$ if $w = f(x(s,t), y(s,t))$, $x = s^2t^3$, $y = 3s^5 + 2\ln\left(t/2\right)$, $f_x(-1,2) = 3$, $f_x(8,-3) = 4$, $f_x(12,1) = 7$, $f_y(-1,2) = -5$, $f_y(8,-3) = -11$, and $f_y(12,1) = -2$, and f_x and f_y are continuous everywhere.

6. Find $g'(1)$ if $g(t) = f(x(t), y(t), z(t))$, $x(t) = 2t^2$, $y(t) = t^3$, and $z(t) = -t^{-2}$, $f_x(2,1,-1) = 3$, $f_x(4,3,2) = -2$, $f_y(2,1,-1) = -5$, $f_y(4,3,2) = -2$, $f_z(2,1,-1) = 7$, and $f_z(4,3,2) = -1$, and all derivatives of f are continuous.

Answers:

1. $w_t = f_x x_t + f_y y_t = -\tan^{-1}(y) 2s \sin(2t) + \frac{2x}{1+y^2}$

2. $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} = 2y^2 r t^3 e^{3z} + 9xy^2 r^2 e^{3z}$

3. $\frac{dt}{ds} = t_x x' + t_y y' + t_z z' + t_w w'$

$$= (e^{xy} + xye^{xy})2s + (x^2 e^{xy} + w^3 z)5 - w^3 y \sin(s) - \frac{6w^2 yz}{s^2}$$

4. $g_t(s,t) = f_x(x(s,t), y(s,t)) x_t(s,t) + f_y(x(s,t), y(s,t)) y_t(s,t)$

$$= f_x(x(s,t), y(s,t)) 3s + f_y(x(s,t), y(s,t))\left(-16st^{-3}\right)$$

$$g_t(1,-2) = f_x(-6,2)3 + f_y(-6,2)2$$

$$= 9$$

5. $\frac{\partial w}{\partial t} = f_x x_t + f_y y_t$

$$= f_x(x(s,t), y(s,t)) 3s^2 t^2 + f_y(x(s,t), y(s,t)) \frac{2}{t}$$

$$\left.\frac{\partial w}{\partial t}\right|_{(s,t)=(-1,2)} = f_x(8,-3)12 + f_y(8,-3)1$$

$$= 37$$

6. $g'(t) = f_x(x(t), y(t), z(t)) x'(t) + f_y(x(t), y(t), z(t)) y'(t) + f_z(x(t), y(t), z(t)) z'(t)$

$$= f_x(x(t), y(t), z(t)) 4t + f_y(x(t), y(t), z(t)) 3t^2 + f_z(x(t), y(t), z(t)) 2t^{-3}$$

$$g'(1) = f_x(2,1,-1)4 + f_y(2,1,-1)3 + f_z(2,1,-1)2$$

$$= 11$$