Exam 5 Math 208 Fall 2004

Name: Key Score:

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is required.

(20) **1.** A vector field is given by $\mathbf{F} = \langle 2y, 3x^2 \rangle$.

(a) Is $\mathbf{F}$ conservative? Justify your answer.

$\underline{F} = \langle M, N \rangle$ with $M = 2y$, $N = 3x^2$.

10 Hence $N_x = 6x$, $M_y = 2$ and $N_x \neq M_y$.

So $\underline{F}$ is <u>not conservative</u>

(14.1.42) (b) Find equations for all flow lines for the vector field $\mathbf{F}$.

Have $dx/dt = 2y$, $dy/dt = 3x^2$, so $dy/dx = \dfrac{dy/dt}{dx/dt} = \dfrac{3x^2}{2y}$.

0 Thus $2y\,dy = 3x^2\,dx$. Integrate: $\int 2y\,dy = \int 3x^2\,dx$

Thus $2\dfrac{y^2}{2} = 3\dfrac{x^3}{3} + C$, ie $\underline{y^2 = x^3 + C}$ -2 if no const of integr.

14.4.41) [redacted] (20) **2.** Let $\mathbf{F} = \frac{1}{x^2+y^2}\langle -y, x \rangle = \nabla \arctan(y/x)$. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the unit circle with center at the origin, oriented clockwise.

C is parametrized by $x = \cos t$, $y = \sin t$, $2\pi \geq t \geq 0$

So $dx = (-\sin t)\,dt$, $dy = (\cos t)\,dt$. 8

Hence $\oint_C \underline{F} \cdot d\underline{r} = \oint_C \dfrac{-y\,dx}{x^2+y^2} + \dfrac{x\,dy}{x^2+y^2}$

12 $= \int_{2\pi}^{0} \dfrac{-\sin t(-\sin t)\,dt + \cos t(\cos t)\,dt}{1^2}$

$= \int_{2\pi}^{0} (\sin^2 t + \cos^2 t)\,dt = -\int_0^{2\pi} 1 \cdot dt = \underline{-2\pi}$

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(14.3.18)(20) **3.** Let $\mathbf{F} = \langle 3x^2y^2, 2x^3y - 4\rangle$. Show that the line integral

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C 3x^2y^2dx + \left(2x^3y - 4\right)dy$$

is path independent and use the fundamental theorem for line integrals to evaluate this integral if C runs from $(1,2)$ to $(-1,1)$. (You should find a potential function.)

Spse $\underline{F} = \nabla f = \langle f_x, f_y\rangle$, so $f_x = 3x^2y^2$, $f_y = 2x^3y - 4$.

Thus $f = \int f_x\,dx = \int 3x^2y^2\,dx = \cancel{3}\frac{x^3}{\cancel{3}}y^2 + C(y)$.

2 So $f_y = 2x^3y + C'(y) = 2x^3y - 4$ so $C'(y) = -4$,

and we can take $C(y) = -4y$; Thus

$f(x,y) = x^3y^2 - 4y$.

Alternate soln: Show $M_y = N_x$, mention simply cnncted domain & use any path from $(1,2)$ to $(-1,1)$ to get line integral

So by FTOLI,

8 $\int_C 3x^2y^2dx + (2x^3y-4)dy = f(-1,1) - f(1,2)$

$= (-1)^3 1^2 - 4\cdot 1 - (1^3\cdot 2^2 - 4\cdot 2) = -5 + 4 = \underline{-1}$

(14.4.27)(20) **4.** The closed curve $x^{2/3} + y^{2/3} = 1$ is parametrized by the equations $x = \cos^3 t$ and $y = \sin^3 t$, where $0 \le t \le 2\pi$. Use this fact and Green's theorem to express the area of the region R bounded by this curve as a definite integral in t. *Do not evaluate the integral.*

We have $x = \cos^3 t$, $y = \sin^3 t$, $0 \le t \le 2\pi$.

So $dx = 3\cos^2 t\,(-\sin t)\,dt$

$dy = 3\sin^2 t\,(\cos t)\,dt$ 4

Also, if we use $\underline{F} = \langle 0, x\rangle$, we get $N_x - M_y = 1 - 0 = 1$.

[other choices: $\frac{1}{2}\langle -y, x\rangle$, $\langle -y, 0\rangle$, etc] 4

So Area of $R = \iint_R 1\cdot dA = \iint_R (N_x - M_y)\,dA = \oint_C x\cdot dy$ 5

$= \int_0^{2\pi} \cos^3 t \cdot 3\sin^2 t\cdot\cos t\,dt$

$= 3\int_0^{2\pi} (\cos^4 t)(\sin^2 t)\,dt$ 5

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(20) **5.** Evaluate

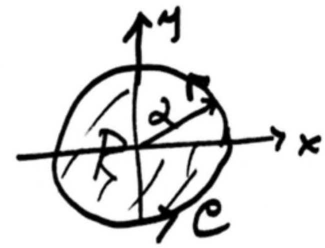
$$\oint_C \left(e^{x^2} - 2y\right) dx + \left(e^{y^2} + 4x\right) dy$$

where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise. (Green's theorem might help here.)

4 Here we set $\underline{F} = \langle e^{x^2} - 2y,\ e^{y^2} + 4x \rangle = \langle M, N \rangle$

Green says integral above is just

5 $$\oint_C \underline{F} \cdot d\underline{r} = \iint_R (N_x - M_y)\, dA$$



where R is interior of circle and here

5 $$N_x = \frac{\partial}{\partial x}\left(e^{y^2} + 4x\right) = 4,$$

$$N_y = \partial/\partial y \left(e^{x^2} - 2y\right) = -2.$$

6 So $$\oint \underline{F} \cdot d\underline{r} = \iint_R (4 - -2)\, dA = 6 \iint_R dA = 6 \cdot \pi \cdot 2^2$$

$$= \underline{\underline{24\pi}}$$