Exam 4 Math 208 **Fall 2004**

Name: Key **Score:**______

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is required.

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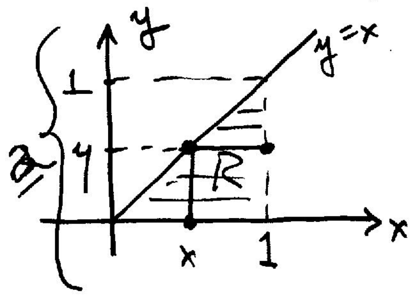
(16) **1.** Reverse the order of integration in the iterated integral $I = \int_0^1 \int_y^1 3xe^{x^3}\,dx\,dy$ and use this to evaluate I exactly.



From region we have

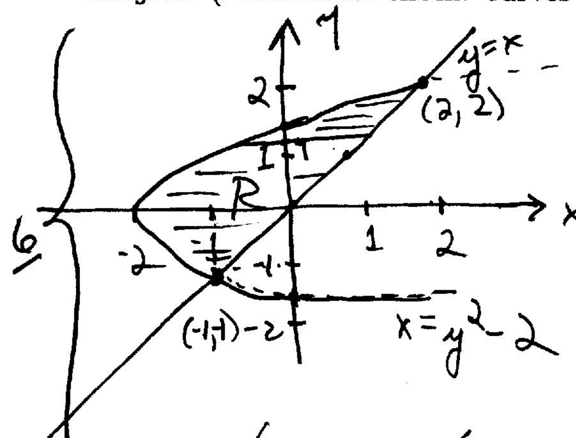
$$I = \iint_R 3xe^{x^3}\,dA = \int_0^1 \int_0^x 3xe^{x^3}\,dy\,dx \quad 7$$

$$3 \quad = \int_0^1 3xe^{x^3}\left(\int_0^x 1\cdot dy\right)dx = \int_0^1 3xe^{x^3}(x-0)\,dx$$

$$= \int_0^1 e^{x^3}\cdot 3x^2\,dx \qquad u = x^3,\ du = 3x^2\,dx \quad u(0)=0 \quad u(1)=1$$

$$4 \quad = \int_0^1 e^u\,du = e^1 - e^0 = \underline{e-1}.$$

(13.2.21) (16) **2.** Express the volume of the solid bounded by the curves $z = x+2$, $z=0$, $x = y^2-2$ and $x=y$ as an interated integral in x, y. Sketch the plane region of integration. *Do not evaluate the integral.* (Arithmetic check: curves $x = y^2 - 2$ and $x = y$ intersect at $(-1,-1)$ and $(2,2)$.)



3: Curves $x = y^2-2$, $x = y$ intersect where $0 = y^2 - x - 2 = x^2 - x - 2 = (x-2)(x+1)$

So $x = 2, -1$, points are $(2,2)$, $(-1,-1)$

Top is $z = x+2$, bottom $z = 0$.

So volume is

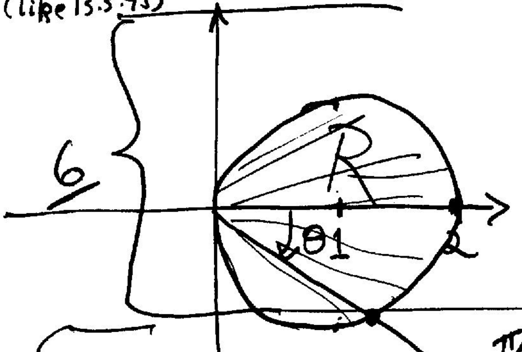
$$V = \iint_R (\text{top} - \text{bottom})\,dA$$

$$7 \quad = \int_{-1}^{2}\int_{y^2-2}^{y} (x+2-0)\,dx\,dy$$

/2

(22) **3.** Calculate the mass of a lamina that occupies the plane region R bounded by the curve $(x-1)^2+y^2=1$ and has density function $\rho(x,y)=1/\sqrt{x^2+y^2}$. Set up an integral for the nonzero moment of this mass (one moment is zero by symmetry), but *do not evaluate the integral.*

(like 13.3.45)



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For R: circle, center $(1,0)$, radius 1.

Here $x^2-2x+1+y^2=1$, so $r^2-2r\cos\theta=0$

Hence $r^2=2r\cos\theta$, so $r=2\cos\theta$.

Now mass is

$$M=\iint_R \frac{1}{\sqrt{x^2+y^2}}\,dA=\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta}\frac{1}{\cancel{r}}\cdot\cancel{r}\,dr\,d\theta=\int_{-\pi/2}^{\pi/2} r\Big|_0^{2\cos\theta}d\theta=\int_{-\pi/2}^{\pi/2}2\cos\theta\,d\theta$$

$$=2\sin\theta\Big|_{-\pi/2}^{\pi/2}=2(1-(-1))=4$$

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By symmetry, $M_x=0$. And

$$M_y=\iint_R x\,\frac{1}{\sqrt{x^2+y^2}}\,dA=\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r\cos\theta\cdot\frac{1}{\cancel{r}}\cancel{r}\,dr\,d\theta$$

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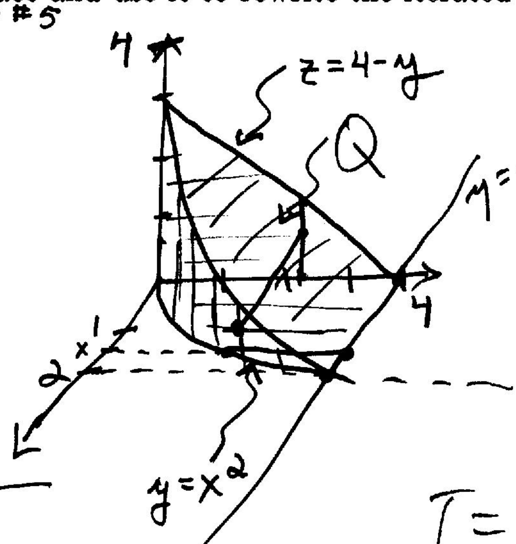
(22) **4.** Let $I=\int_0^2\int_{x^2}^4\int_0^{4-y}(x+yz)\,dz\,dy\,dx$ Sketch the solid over which this integration takes place and use it to rewrite the iterated integral in the order $dx\,dz\,dy$. *Do not evaluate the integral.*

Smp. Ex #5



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From integral we see Q is bd by $z=4-y$ (top), $z=0$ (bottom), sides $x=0$, $y=x^2$.

In other order

$$I=\int_0^4\int_{z=0}^{4-y}\int_{x=0}^{\sqrt{y}}(x+yz)\,dx\,dz\,dy$$

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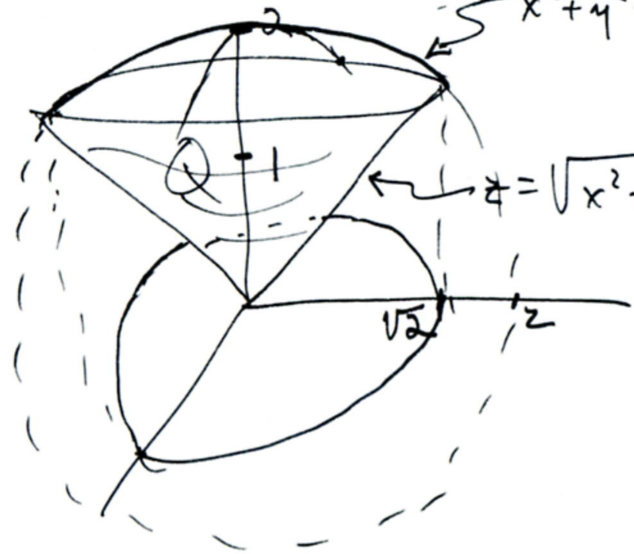
(13.8.54)(24) **5.** Let Q be the region between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{4 - x^2 - y^2}$.

(a) Sketch this region.



$x^2+y^2+z^2 = 4 = 2^2$, OR: $z = \sqrt{4-r^2}$

$z = \sqrt{x^2+y^2} = r$

This is "ice-cream cone"

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Intersect at $2(x^2+y^2) = 4$

$x^2+y^2 = 2 = (\sqrt{2})^2$

(b) Set up the triple integral $I = \iiint_Q z\, dV$ in cylindrical coordinates. *Do not evaluate the integral.*

$$I = \int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} z\, r\, dz\, dr\, d\theta$$

−2 per lim. err

needed here. 3 pts for resulting lim.

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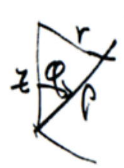
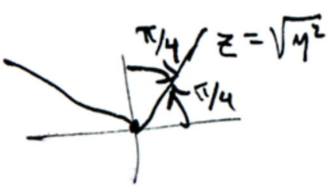
(c) Set up the triple integral $I = \iiint_Q z\, dV$ in spherical coordinates. *Do not evaluate the integral.*

−2 if no z conv.

Rcl That $z = \rho\cos\varphi$, $r = \rho\sin\varphi$. So



$$I = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho\cos\varphi \cdot \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$



$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho^3\cos\varphi\sin\varphi\, d\rho\, d\varphi\, d\theta$$

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