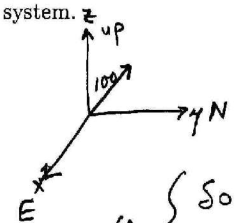
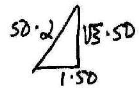
Exam 2 **Math 208** **Fall 2004**

Name: Key **Score:**________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is required.

(30) **1.** A projectile is launched on level ground due north from ground level at an angle of $\pi/6$ above the horizontal with an initial speed of 100 ft/sec. The only force on the projectile is gravity.
(a) Find the acceleration, velocity and position as functions of time in a three dimensional coordinate system.





We have $\underline{r}(0) = \langle 0,0,0\rangle$, $\underline{v}(0) = \langle 0, 50, 50\sqrt{3}\rangle$ } 2

$\underline{a} = \langle 0,0,-32\rangle$ } 4

4 { So $\underline{v}(t) = \int \underline{a}\,dt = \langle 0,0,-32t\rangle + \underline{C}$, $\underline{v}(0) = \underline{C} = \langle 0,50,50\sqrt{3}\rangle$,
So $\underline{v}(t) = \langle 0, 50, 50\sqrt{3} - 32t\rangle$

4 { So $\underline{r}(t) = \int \underline{v}(t)\,dt = \langle 0, 50t, 50\sqrt{3}t - 16t^2\rangle + \underline{D}$
and $\langle 0,0,0\rangle = \underline{r}(0) = \langle 0,0,0\rangle + \underline{D}$, so $\underline{D} = \underline{0}$ and
$\underline{r}(t) = \langle 0, 50t, 50\sqrt{3}t - 16t^2\rangle$

(b) Use (a) to find the time to impact and the velocity of the projectile at impact.

4 { Impact occurs when z-coordinate of $\underline{r}$ is 0, i.e.
$0 = 50\sqrt{3}t - 16t^2 = (50\sqrt{3} - 16t)t$. So $t = 0$ or
$50\sqrt{3} - 16t = 0$,
$t = \frac{50}{16}\sqrt{3} = \frac{25}{8}\sqrt{3}$

4 { At impact, velocity is
$\underline{v}\left(\frac{25}{8}\sqrt{3}\right) = \langle 0, 50, 50\sqrt{3} - \cancel{32}^{4}\cdot\frac{25}{\cancel{8}}\sqrt{3}\rangle = \langle 0, 50, -50\sqrt{3}\rangle$

(c) Suppose that the projectile as in (a) is launched into a steady wind which blows the projectile due east at 20 ~~miles per hour~~ feet per second. What are the new position, velocity and acceleration vectors for the projectile?

In this case there is no new force. So
$\underline{a} = \langle 0,0,-32\rangle$ is unchanged. } 2

New velocity is $\underline{v}(t) = \underline{v}_{old}(t) + \langle 20,0,0\rangle$
$= \langle 20, 50, 50\sqrt{3} - 32t\rangle$ } 3

New position is $\underline{r}(t) = \int \underline{v}(t)\,dt = \langle 20t, 50t, 50\sqrt{3}t - 16t^2\rangle + \underline{D}$
But $\underline{D} = \underline{r}(0) = \underline{0}$, so $\underline{r}(t) = \langle 20t, 50t, 50\sqrt{3}t - 16t^2\rangle$ } 2

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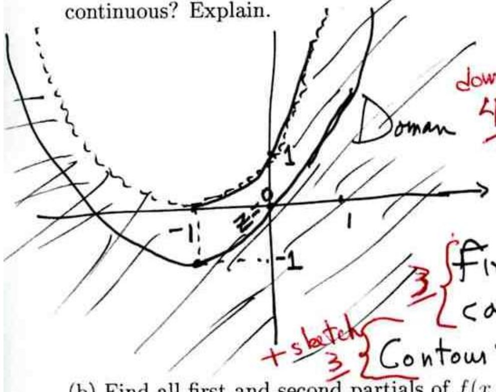
(30) **2.** A function is defined by $f(x,y) = \ln(x^2 + 2x + 1 - y)$.

(a) Sketch the domain of $f(x,y)$ and one contour of $f(x,y)$ in the xy-plane. Where is $f(x,y)$ continuous? Explain.



domain + sketch 4

For $f(x,y)$ to be defined, $x^2+2x+1-y > 0$

Ie $y < (x^2+2x+1) = (x+1)^2$

Domain of f is $\{(x,y) \mid y < (x+1)^2\}$.

3 $f(x,y)$ is alg. expression, so by Key Fact is continuous wherever defined.

+ sketch 3 Contour: $z = 0 = \ln(x^2+2x+1-y)$, so $1 = e^0 = (x+1)^2 - y$

Ie $y = (x+1)^2 - 1$

(b) Find all first and second partials of $f(x,y)$. [Do not simplify]

2 $f_x = \dfrac{1\cdot(2x+2)}{x^2+2x+1-y}$ 2 $f_y = \dfrac{-1}{x^2+2x+1-y}$

2 $f_{xx} = \dfrac{(x^2+2x+1-y)(2)-(2x+2)(2x+2)}{(x^2+2x+1-y)^2}$

2 $f_{yy} = \dfrac{-1}{(x^2+2x+1-y)^2}$

2 $f_{yx} = \dfrac{1\cdot(2x+2)}{(x^2+2x+1-y)^2}$ and $f_{yx} = f_{xy}$ 2 (or calculate it)

(c) Find an equation for the tangent plane to the surface $z = f(x,y)$ at the point $(0,0,0)$.

Plane is $z = 0 + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

So $z = \frac{2}{1}x + (-1)y$

$z = 2x - y$

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(10) **3.** Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y}{x^2+y^2}$ does not exist.

Use polar coordinates:

$\dfrac{x^2-y}{x^2+y^2} = \dfrac{r^2\cos^2\theta - r\sin\theta}{r^2}$

$= \cos^2\theta - \frac{1}{r}\sin\theta$. 5

so if $\sin\theta \neq 0$, This is unbounded as $r \to 0$ and limit DNE 5

OR: Use $y=0$ (x-axis) & approach 0:

$\frac{x^2}{x^2} = 1 \xrightarrow[(x,y)\to(0,0)]{} 1$ 5

Use $x=0$ (y-axis) and approach 0

$-\frac{y}{y^2} = -\frac{1}{y}$ which is unbounded as $y \to 0$. So limit DNE 5

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(20) **4.** Let $z = f(x, y) = x^2 + y^2 - 1$.
(a) Find the total differential dz for $f(x, y)$ at the point $(2, 1)$.

(4 pts for derivs)

In general $dz = f_x(x,y)\,dx + f_y(x,y)\,dy$
$= 2x\,dx + 2y\,dy.$ 7

When $x=2, y=1$, $dz = 4\,dx + 2\,dy$ 3

(b) Use differentials to estimate the maximum variance of $f(x, y)$ from $f(2, 1)$ (this means $|f(x, y) - f(2, 1)|$) given that $|x - 2| \leq 0.2$ and $|y - 1| \leq 0.1$.

Take $x = 2 + dx$, so $x - 2 = dx$ and $|dx| \leq 0.2$
$y = 1 + dy$, so $y - 1 = dy$ and $|dy| \leq 0.1$ 3

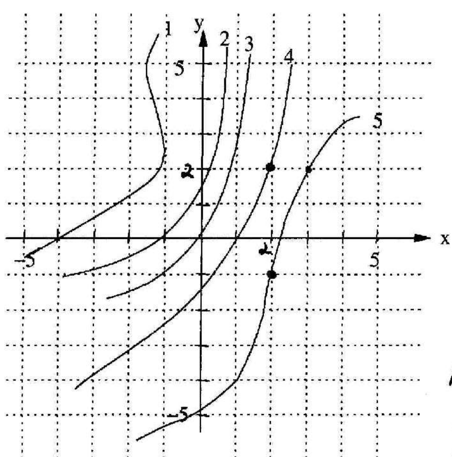
Thus $|f(x,y) - f(2,1)| = |f(2+dx, 1+dy) - f(2,1)| = |\Delta z| \approx |dz|$ 3

But from (a)

4 $|dz| = |4\,dx + 2\,dy| \leq 4|dx| + 2|dy| \leq 4 \cdot 0.2 + 2 \cdot 0.1$
$\leq 0.8 + 0.2 = \underline{\underline{1.0}}$

−1 if correct range but no abs val. arg, unless pos vals. indicat

(10) **5.** A smooth function $f(x, y)$ (this means f and its first partials are continuous) has the following contour graph. Use the contour graph to estimate $\partial f/\partial x$ and $\partial f/\partial y$ at $(2, 2)$.



$f_x \approx \dfrac{f(2+h, 2) - f(2,2)}{h}$ 1

From the pic., $f(2,2) \approx 4$ 2
$f(3,2) \approx 5$ 1

Take $h = 1$ and get
$f_x(2,2) \approx \dfrac{5-4}{1} = 1$ 2

Also, $f_y \approx \dfrac{f(2, 2+h) - f(2,2)}{h}$ 1

From the pic., $f(2,-1) \approx 5$ 2

(−5 for using origin Eqtty cond. ~~...~~)

So take $h = -3$ and get
$f_y(2,2) \approx \dfrac{5-4}{-3} = \underline{\underline{-1/3}}$ 1

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