Exam 3 **Linear Algebra** **Sample**

Name:______________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculators are allowed. Unless otherwise stated, exact answers are required (e.g., π rather than 3.1415).

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(25) **1.** Let $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (-1, 1, 1)$, $\mathbf{v_3} = (1/2, -1/2, 1)$ and $\mathbf{v} = (1, 2, -2)$.

(a) Find the norm of $\mathbf{v}$.

(b) Find the cosine of the angle between the vectors $\mathbf{v}$ and $\mathbf{v}_1$.

(c) Verify the CBS inequality for the pair of vectors $\mathbf{v}, \mathbf{v}_2$.

(d) Show $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v_3}$ is an orthogonal set, hence a basis of $\mathbb{R}^3$. Why a basis (no proofs, just reasons)?

(e) Find the coordinates of $\mathbf{v}$ relative to this basis.

(15) **2.** Display and solve the normal equations for the system below. Are the solution(s) you found genuine solutions to the original system?

$$2x_1 - 2x_2 = 2, \quad x_1 + x_2 = -1, \quad 3x + x_2 = 1$$

(15) **3.** Find an eigensystem for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

(20) **4.** Assume that eigenvalues of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ are $0, 5$ and two eigenvectors are $\mathbf{v}_1 = (-2, 1)$ and $\mathbf{v}_2 = (1, 2)$.

(a) Use the given information to find a formula for $\mathbf{x}^{(k)}$, given that $\mathbf{x}^{(0)} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$ and $\mathbf{x}^{(j+1)} = A\mathbf{x}^{(j)}$ for all j.

(b) Use the given information to diagonalize A. (You only need exhibit the diagonal matrix D and diagonalizing matrix P.)

(c) Use (b) to find a formula for powers of A in terms of powers of eigenvalues of A.

(16) **5.** Answer True/False (T/F), fill in the blanks and if two words are in italics and separated by a slash, underline the correct word. Each correct answer is worth 2 points, false answer worth -1 points and blank answer worth 0 points for a minimum of 0 and maximum of 16:

(a) If A is a real matrix, then $A^T A$ is digaonalizable (T/F) ______________.

(b) If A is a nonzero real matrix, then $A^T A$ is positive definite (T/F) ______________.

(c) Eigenvectors of a matrix cannot be zero (T/F) ______________.

(d). If Q is an $n \times n$ orthogonal matrix and $\mathbf{v} \in \mathbb{R}^n$, then $\|Qv\| = \|v\|$ (T/F)______________.

(e) If $\rho(A) < 1$ and $\mathbf{x}^{(k+1)} = A\mathbf{x}^{(k)}$, then $\lim_{k\to\infty} \mathbf{x}^{(k)}$ equals ______________________________.

(f) The matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1+i & i \\ i & 1-i \end{bmatrix}$ is *unitary* / *non-unitary* because ______________________________

(g) The matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is *diagonalizable* / *non-diagonalizable* because ______________________________

(h) The matrix $\begin{bmatrix} 1 & i \\ -i & 3 \end{bmatrix}$ is *diagonalizable* / *non-diagonalizable* because ______________________________

(9) **6.** Prove **one and only one** of the following:

(a) If A is invertible and λ is an eigenvalue of A, then $1/\lambda$ is an eigenvalue of A^{-1}.

(b) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v_3}$ is an orthogonal set of nonzero vectors, then this set is linearly independent.