Name:____

Score:__

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, but clearly so indicate. You must clearly identify answers and show supporting work to receive any credit. Point values of problems are given in parentheses. Notes or text in any form not allowed. Calculators are allowed.

- (30) **1.** Let $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4] = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \end{bmatrix}$ with reduced row echelon form $R = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
- (a) Find a basis for $\mathcal{R}(A)$, the row space of A.
- (b) Find a basis for C(A), the column space of A.

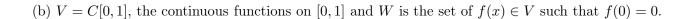
(c) Find a basis for $\mathcal{N}(A)$, the null space of A.

(d) Find all possible linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ that sum to $\mathbf{0}$.

(e) Which \mathbf{v}_j 's are redundant in the list of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$?

(f) Find a basis of \mathbb{R}^3 containing a basis of $\mathcal{C}(A)$.





(c)
$$W = \{ \mathbf{v} \in V \mid T(\mathbf{v}) = 0 \}$$
 where $T : V \to U$ is a linear operator (you may assume that $T(0) = 0$.)

(10) **3.** Show that 1 + x, $x + x^2$, 1 - x is a basis of \mathcal{P}_2 , the space of polynomials of degree at most two, and find the coordinates of $2 + x^2$ relative to this basis.

(15) 4. You are given that $\mathbf{w}_1 = (0, 1, 0)$, $\mathbf{w}_2 = (1, 1, 1)$ is a linearly independent set in $V = \mathbb{R}^3$ and $\mathbf{v}_1 = (2, -1, 1)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (1, 3, 1)$ a basis of V . Steinitz substitution says that \mathbf{w}_1 , \mathbf{w}_2 can be substituted into the basis in place of certain \mathbf{v}_i 's. Which substitutions work?
(14) 5. Fill in the blanks or answer True/False (T/F). Each correct answer is worth 2 points, false answer worth -1 points and blank answer worth 0 points for a minimum of 0 and maximum of 14:
(a) Every vector space is finite dimensional (T/F)
(b) Elementary row operations on a matrix do not change the column space (T/F)
(c) If $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ are both vector solutions to the linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_0$ is in the null space of A . (T/F)
(d) The function $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x,y)) = (x+y,x-2y)$ is linear (T/F)
(e) The Basis Theorem asserts that every finite dimensional vector space
(f) The Dimension Theorem asserts that
(g) A basis of a vector space is by definition
(10) 6. Let U and V be subspaces of the finite dimensional vector space W such that $U \cap V = \{0\}$. Show that $\dim(U+V) = \dim U + \dim V$. (Hint: show the union of bases of U and V is a basis of $U+V$.)