Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculators are allowed.

(24) **1.** Consider the linear system $A\mathbf{x} = \mathbf{b}$ with $\mathbf{b} = (2, 1, 4)$ given by the following:

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 2 \\ 2x_1 + x_2 - 2x_4 &= 1 \\ 2x_1 + 2x_2 + 2x_3 - 2x_4 &= 4 \end{aligned}$$

(a) Use Gauss-Jordan elimination to find the general solution to this system. Clearly specify the elementary row operations you use.

(b) What is the coefficient matrix A of this system and its rank and nullity?

(c) Apply the row operations used in part (a) in the same order as was used in (a) to a general right hand side vector $\mathbf{b} = (b_1, b_2, b_3)$. What is the resulting vector?

(d) Use (c) to to find a condition on $\mathbf{b}$ that ensures that the system $A\mathbf{x} = \mathbf{b}$ is consistent.

(14) **2.** Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$ and find the inverse of A. Use this inverse to solve $A\mathbf{x} = \mathbf{b}$ with $\mathbf{b} = (1, -1, 1)$.

(10) **3.** Solve the following systems for the (complex) variable z. Express your answere in standard form ($z = x + iy$) where possible.

(a) $(2 + i)z = 1$

(b) $z^4 = 1$

(20) **4.** Carry out these calculations or indicate they are impossible. You are given that $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1+i \\ 0 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$,

(a) $\mathbf{y}C\mathbf{x}$

(b) $\mathbf{x}\mathbf{y}$

(c) $\mathbf{x} + 2\mathbf{x}^T$

(d) C^H

(e) C^{-1}

(f) CD

(18) **5.** Fill in the blanks or answer True/False (T/F). Each correct answer is worth 2 points, false answer worth -1 points and blank answer worth 0 points for a minimum of 0 and maximum of 18:
(a) If A is a 2×2 nonzero matrix and the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for some $\mathbf{b}$ then the rank of A is ________________

(b) Function $T((x, y)) = (x + y, 2x, 4y - x)$ is a matrix multiplication function $T_A((x, y))$, where $A =$

(c) For 3×3 matrices, $\det E_i(-2) =$________________, $\det E_{ij} =$________________ and $\det E_{ij}(3) =$________________.

(d) If A is a 4×4 matrix, then in terms of $\det A$, we have $\det(-3A) =$________________.

(e) As a matrix product $x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} =$

(f) Any homogeneous (right hand side vector 0) linear system is consistent (T/F):

(g) If A, B are 2×2 matrices, then $(AB)^2 = A^2B^2$ (T/F):

(h) Every diagonal matrix is symmetric (T/F):

(i) If A is invertible, then $\operatorname{rank} AB = \operatorname{rank} B$ (T/F):

(14) **6.** Let A be an $n \times n$ matrix.
(a) Define what it means for A to be invertible.

(b) Prove from definition that the product AB of two invertible matrices A and B is itself invertible.

(c) Show by example that the sum of two invertible matrices need not be invertible.