JDEP 384H: Numerical Methods in Business

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110 Kaufmann Center

Outline

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A Model Problem

The Problem:

Dominant strategy elimination and the more general maximin/minimax strategies will not solve the following problem.

Strategy		Player 2: 1	2	3
Player 1	1	2	3	-2
	2	-1	4	0
	3	3	-2	-1

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Strategies

Solution:

Use *mixed strategies* instead of pure strategies, i.e., a probability vector (x_1, x_2, x_3) for player 1 ($y = (y_1, y_2, y_3)$ for player 2) that maximizes for player 1 (minimizes for player 2) the payoff for all possible plays by the opponent.

- If the payoff table is converted into a matrix A ($m \times n$ in general), then the payoff for any pair of mixed strategies is

$$p = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_i y_j = \mathbf{x}^T A \mathbf{y}$$

- Player 1's goal: Find probability vector $\mathbf{x}$ solving $\max_{\mathbf{x}} \min_{\mathbf{y}} \mathbf{x}^T A \mathbf{y}$.
- Player 2's goal: Find probability vector $\mathbf{y}$ solving $\min_{\mathbf{y}} \max_{\mathbf{x}} \mathbf{x}^T A \mathbf{y}$.

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Strategies

Linear Programming to the Rescue:

We can solve either problem of the previous frame with linear programming tools as follows.

- **Key fact**: Both problems have a solution with common payoff p. In fact, they solve the linear programming progams max p subject to constraints $\mathbf{x}^T A \geq p\mathbf{1}_{1,n}$, $\mathbf{x} \geq \mathbf{0}$, $\mathbf{x}^T \mathbf{1}_{m,1} = 1$, for player 1 and min p subject to constraints $A\mathbf{y} \leq p\mathbf{1}_{m,1}$, $\mathbf{y} \geq \mathbf{0}$, $\mathbf{1}_{1,n}\mathbf{y} = 1$, for player 2. These linear programming problems are *dual* to each other.
- **Practical tip**: We can also insure that $p \geq 0$ by simply adding a constant to every payoff so that the table is nonnegative, computing the strategies and then subtracting the constant from the computed optimal payoff.
- Let's set up all three examples as LP problems, both explicitly and in matrix form, and solve them with Matlab to determine optimal strategies for each game. Check requirements of `linprog` first.

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A Sensitivity Analysis

The Original Model Problem:

Two companies compete for the bulk of a shared market for a certain product and plan to execute one of three strategies. Both marketing departments analyzed them and have arrived at essentially the same figures for outcomes.

- The three strategies are:
  - Better packaging.
  - An advertising campaign.
  - Slight price reduction.
- Suppose there is considerable uncertainty about the payoff in the case that both players make a slight reduction in price. How could we clearly illustrate the effect of changes in the payoff on the weight that one of the companies puts on this strategy?

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