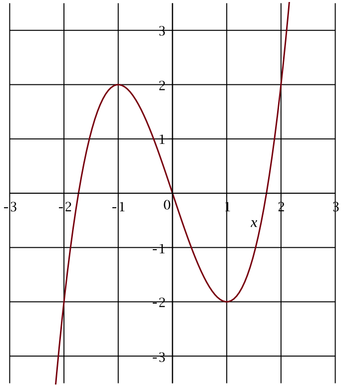
Derivative Function

September 22, 2013

Finding the derivative of a function given graphically

Estimate of function $f(x)$ graph in the Figure at $x = -1, 0, 1$.



Derivative function

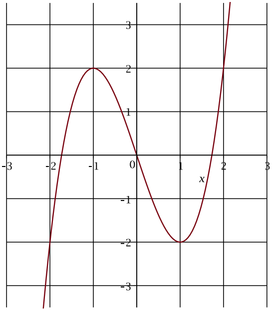
Definition

For a function f, we define the derivative function, f', by

$$f'(x) = \text{Instantaneous rate of change of } f \text{ at } x.$$

Example

Plot the values of the derivative function calculated in the previous example. Compare the graph of f' and f.



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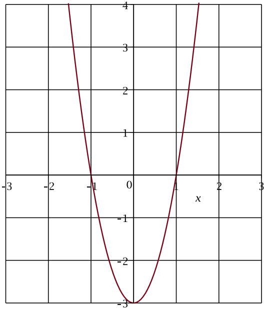
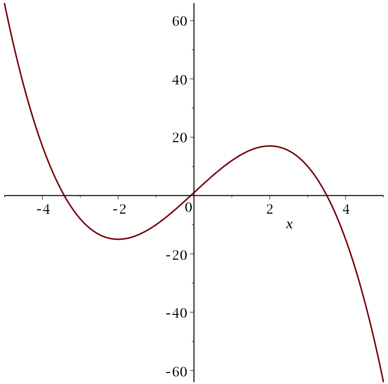


Example

Given the graph of function f in the figure.



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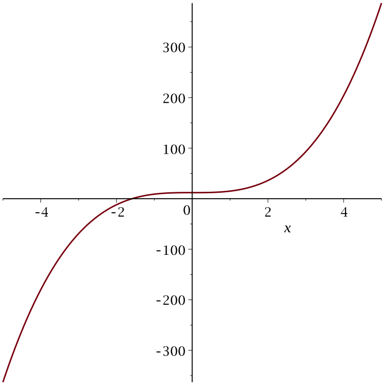
Which of the following graphs is a graph of the derivative, f'?



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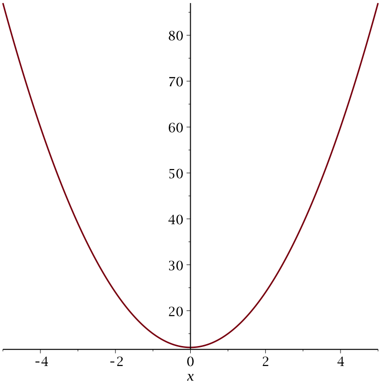

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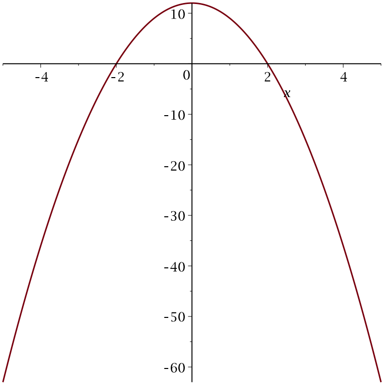

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- If $f' = 0$ every where, then the tangent is horizontal every where, and so f is constant.

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- *If $f' = 0$ on an interval, then f is constant over that interval.*

Estimating the derivative of a function given numerically

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
c(t)	0.84	0.89	0.94	0.98	1	1	0.97	0.90	0.79	0.63	0.41

- The table gives values of $c(t)$, the concentration (mg/cc) of a drug in the bloodstream at time t (min). Construct a table of estimated values for $c'(t)$.

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- $$c'(0.2) \approx \frac{1}{2}(\text{Slope to left of } 0.2 + \text{Slope to right of } 0.2)$$

Example

- Estimate $f'(1)$, $f'(2)$, $f'(3)$ if $f(x) = x^4$.