Local Maxima and Minima

October 22, 2013

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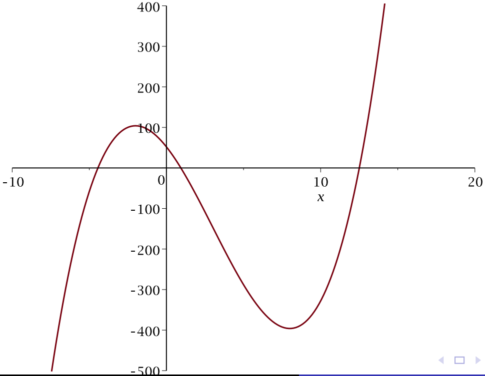

Local maxima and minima

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How do we detect a local maximum or minimum

Definition (Critical point)

For any function f, a point p in the domain of f, where $f'(p) = 0$ or $f'(x)$ is undefined is called a **critical point** of the function. In addition, the point $(p, f(p))$ on the graph of f is also called a critical point (of the graph). A **critical value** of f is the value, $f(p)$, of the function at a critical point, p.

- At a critical point where $f'(p) = 0$, the tangent line to the graph at p is horizontal.

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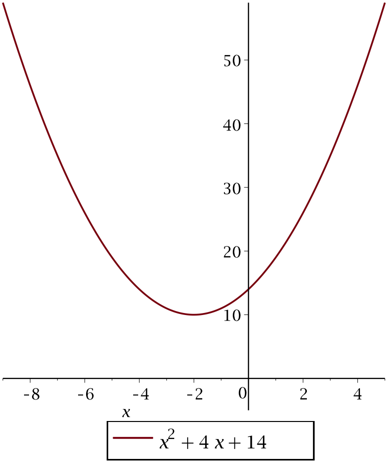
- At a critical point where $f'(p) = 0$, the tangent line to the graph at p is horizontal.
- At a critical point where $f'(p)$ is undefined, there is no horizontal tangent– there is either a vertical tangent or no tangent at all.

Example



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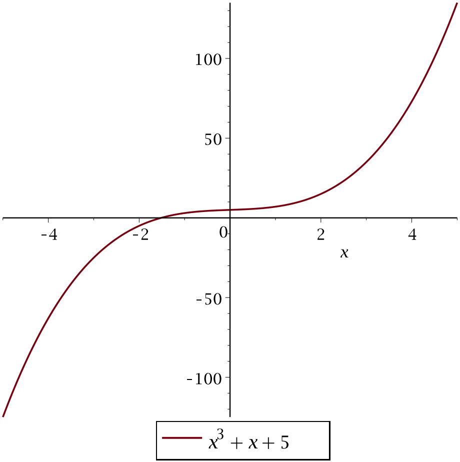

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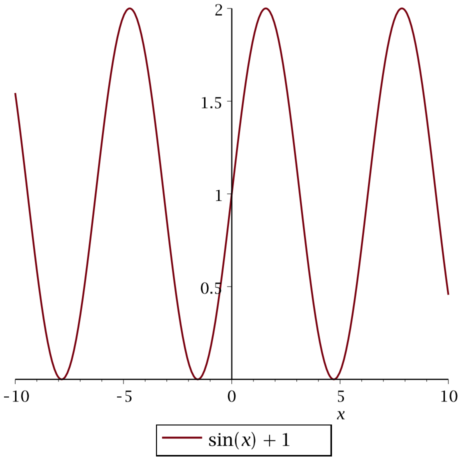

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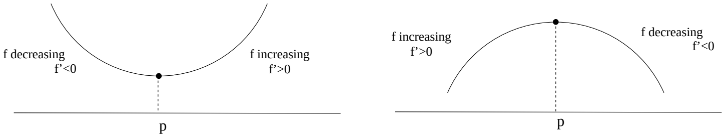
- The critical points divide the domain of f into intervals on which the sign of the derivative remains the same.
- Therefore, if f is defined on the interval between two successive critical points, its graph cannot change direction on that interval, it is either going up or it is going down.
- If a function, continuous on an interval (its domain), has local maximum or minimum at p, then p is a critical point or an endpoint of the interval.

First Derivative Test for Local Maxima and Minima

Suppose p is a critical point of a continuous function f. Then, as we go from left to right:

- If f changes from decreasing to increasing at p, then f has a local minimum at p.



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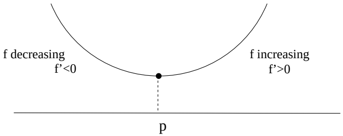
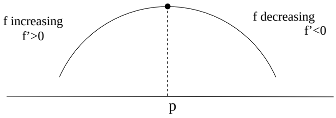
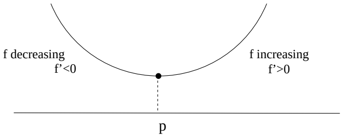


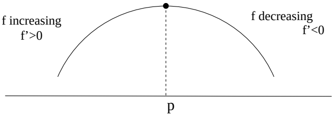


First Derivative Test for Local Maxima and Minima

Suppose p is a critical point of a continuous function f. Then, as we go from left to right:

- If f' changes from negative to positive at p, then f has a local minimum at p.
- If f' changes from positive to negative at p, then f has a local maximum at p.





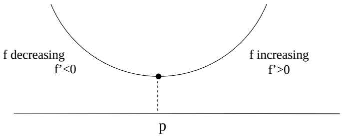
Second Derivative Test for Local Maxima and Minima

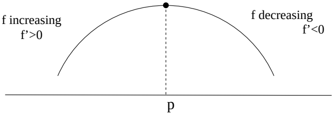
Suppose p is a critical point of a continuous function f, and $f'(p) = 0$.

- If f is concave up at p, then f has a local minimum at p.

equivalent to

- If $f''(p) > 0$, then f has a local minimum at p.





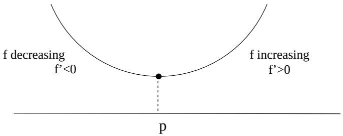
Second Derivative Test for Local Maxima and Minima

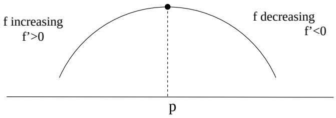
Suppose p is a critical point of a continuous function f, and $f'(p) = 0$.

- If f is concave up at p, then f has a local minimum at p.
- If f is concave down at p, then f has a local maximum at p.

equivalent to

- If $f''(p) > 0$, then f has a local minimum at p.
- If $f''(p) < 0$, then f has a local maximum at p.





Example

(a) Graph a function f with the following properties:

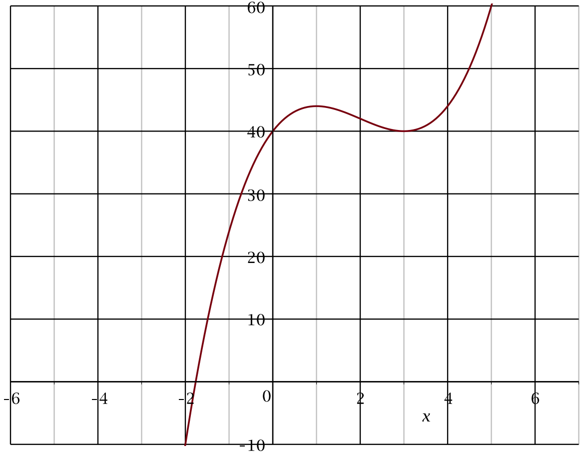
- $f(x)$ has critical point at $x = -2$ and $x = 3$.
- $f'(x)$ is positive on the left of -2 and on the right of 3.
- $f'(x)$ is negative between -2 and 3.

(b) Identify the critical points as local maxima, local maxima, or neither.

Example

Find the local maxima and local minima of $f(x) = x^3 - 6x^2 + 9x + 40$.

Example

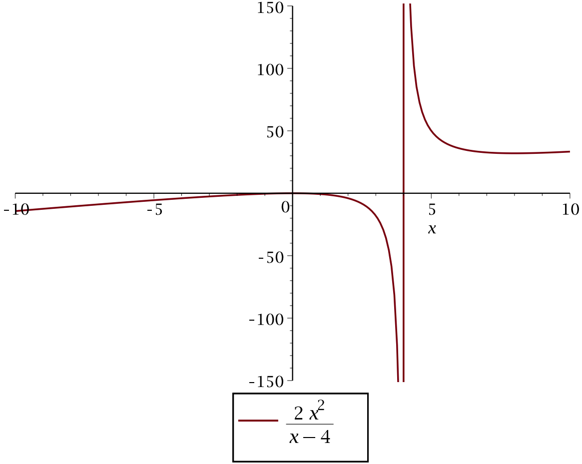


Example

If a and b are nonzero constants, find the domain and all critical points of

$$f(x) = \frac{ax^2}{x - b}.$$

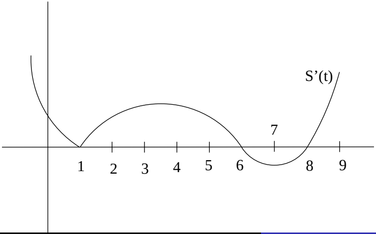
Example



Example

The value of an investment at tiem t is given by $S(t)$. The rate of change, $S'(t)$, of the value of the investment is shown in the figure.

1. What is the critical point of $S(t)$?



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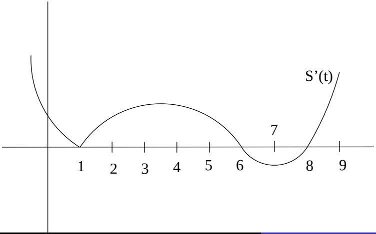

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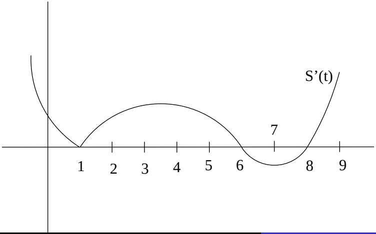
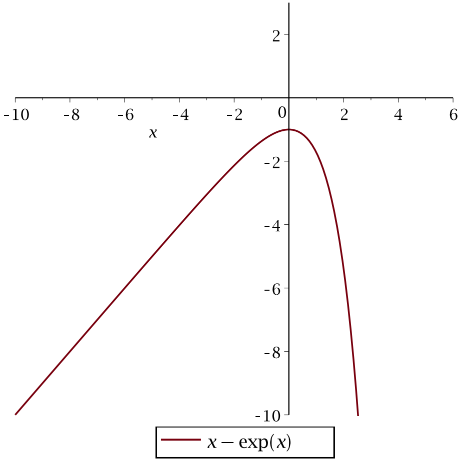
1. What is the critical point of $S(t)$?
2. Identify each critical point as a local maximum, a local minimum, or neither.
3. Explain the financial significance of each of the critical point.



Example

Let $g(x) = x - ke^x$, where k is a constant. For what values of k does the function g have a critical point? a local maximum? a local minimum?

Example



Example