Instantaneous Rate of Change

September 19, 2013

Instantaneous Velocity

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- What is the *exact* velocity at $t = 1$? How can we find it?

Instantaneous Velocity

t	0	0.9	0.99	0.999	1	1.001	1.01	1.1	2
y=s(t)	6.000	83.040	89.318	89.932	90.000	90.068	90.678	96.640	150.000

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Definition of instantaneous velocity using the idea of a limit

When we take smaller interval near $t = 1$, it turns out that the average velocities for the ball are always just above or just below 68 ft/sec. It seems natural to define velocity at the instant $t = 1$ to be 68 ft/sec. It is called the instantaneous velocity at this point. Its definition depends on our being convinced that smaller and smaller intervals provide average velocities that come arbitrarily close to 68. This process is referred to as *taking the limit*.

Definition

The **instantaneous velocity** of an object at time t is defined to be the limit of the average velocity of the object over shorter and shorter time intervals containing t.

Instantaneous rate of change

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The **instantaneous rate of change** of function f at a, also called **rate of change** of f at a, is defined to be the limit of the average rates of change of f over shorter and shorter time intervals around a.

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- The quantity (in milligrams) of a drug in the bloodstream at time t (in minutes) is given by $Q = 25(0.8)^t$. Estimate the rate of change of the quantity at $t = 3$ and interpret our answer.

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- Units of $\frac{\Delta Q}{\Delta t}$ are mg/minute. Since the rate is negative, the quantity of drug is decreasing. After 3 minutes the quantity of the drug in the bloodstream is decreasing at 2.3041 mg/minute.

Derivative at a point

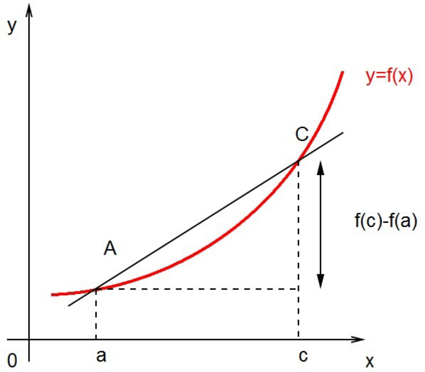
Definition

The **derivative of f at a**, written $f'(a)$, is defined to be the instantaneous rate of change of f at the point a.

Example

- Estimate $f'(2)$ if $f(x) = x^4$.

Visualizing the derivative



- The line passing through A and C is called secant line between $x = a$ and $x = c$.

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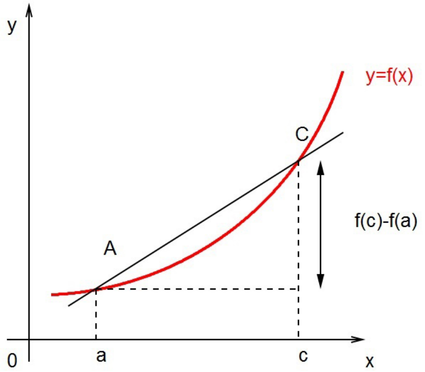

- The line passing through A and C is called secant line between $x = a$ and $x = c$.
- The average rate of change is represented by the slope of the secant line.

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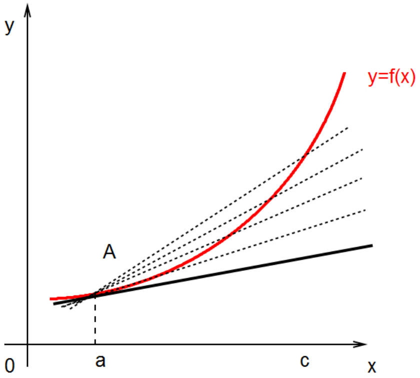

- Move point C toward A, the secant line becomes the tangent line at point A.

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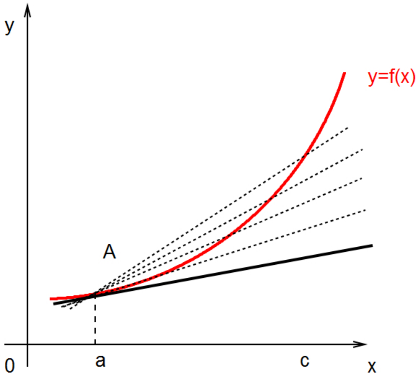

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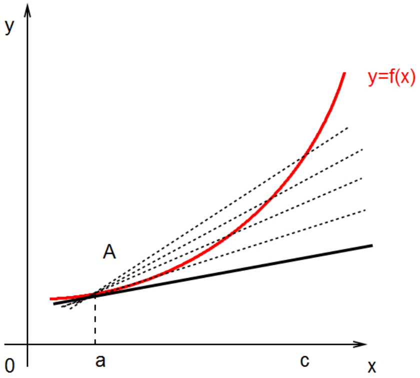

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- The derivative is represented by the slope of the tangent line to the graph at the point.
- We call this slope *the slope of the graph*.

Graph of power functions: $y = x^2$

Let $f(x) = x^2$, determine whether each of the following quantities is positive or negative: $f'(1)$, $f'(0)$, $f'(-2)$, $f'(-1)$.

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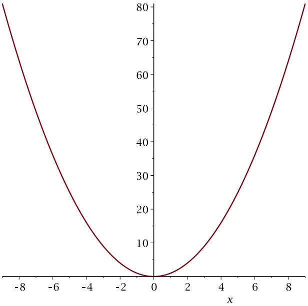

Estimate the derivative of $f(x) = 3^x$ at $x = 0$ graphically and numerically.

Estimate the Derivative of a function given numerically

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Farm land	1039	1012	991	963	945

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- (b) Estimate $f'(1995)$ and interpret your answer in terms of farm land.

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- (a) -4.7
- (b) Estimate $f'(1995)$ and interpret your answer in terms of farm land.

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- (a) -4.7
- (b) $f'(1995) \approx -3.6$.
- In 1995, the amount of farm land was decreaseing at a rate of approximately 3.6 million acres per year.