Construction Antiderivatives Analytically

November 19, 2013

Antiderivative

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If the derivative of $F(x)$ is $f(x)$, then we call $F(x)$ an **antiderivative** of $f(x)$.

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- $e^x + 2$, $e^x + 3$, $e^x + 4$ are antiderivatives of e^x.
- If $F(x)$ is an antiderivative of $f(x)$, then all antiderivatives of $f(x)$ are of the form $F(x) + C$, for some constant C.

Indefinite Integral

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$$\int f(x)dx$$

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- Definite integral, $\int_a^b f(x)dx$, is a **number**; Indefinite integral, $\int f(x)dx$ is a **family of functions**.

Basic Properties of Antiderivatives

Theorem

1. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$
2. $\int c \cdot f(x)dx = c \int f(x)dx$
3. $\int (af(x) + bg(x))dx = a \int f(x)dx + b \int g(x)dx$

Finding antiderivatives

Theorem

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

Finding antiderivatives

Corollary

$$\int k dx = kx + C, \quad for\ any\ constant\ k$$

Finding antiderivatives

Theorem

$$\int \frac{1}{x} dx = \ln|x| + C$$

Examples

Find the following indefinite integrals:

1. $\int (4x^3 + 2x + 1)dx$
2. $\int \frac{x^2+2x^5+2}{x} dx$

Examples

$$\int \left(x^2 + \frac{1}{\sqrt[3]{x}} \right) dx = ?$$

Finding antiderivatives

Theorem

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C, \qquad \textit{for } k \neq 0$$

Corollary

$$\int e^{x}dx = e^{x} + C$$

Examples

$$\int a^{kx} dx = ?$$

Examples

$$\int 14e^{0.24x}dx = ?$$

Examples

$$\int 14xe^{x^2}\,dx = ?$$

Finding antiderivatives

Theorem

$$\int \cos(kx)dx = \frac{1}{k}\sin(kx) + C, \qquad \textit{for } k \neq 0$$

Theorem

$$\int \sin(kx)dx = -\frac{1}{k}\cos(kx) + C, \qquad \textit{for } k \neq 0$$

Examples

$$\int (\sin x + 3\cos(6x))dx = ?$$

Examples

$$\int (\sin(2x) + 100e^{-0.02x})dx = ?$$

Find an antiderivatives $F(x)$ of $f(x)$ so that $F(1) = 4$:

$$f(x) = \frac{-2x^6 + x^3}{x^4}$$

Examples

$$\int 4x^2 \sin(x^3 + 5)dx = ?$$