Second derivative

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- which means $\dfrac{d}{dx}\left(\frac{dy}{dx}\right)$, i.e. the derivative of $\dfrac{dy}{dx}$.

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- *If* $f'' < 0$ *on an interval, then* f' *is decreasing over that interval, so the graph of* f *is concave down there.*

Interpretation of the second derivative as a rate of change

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- If the second derivative is positive, the rate of change is increasing; if the second derivative is negative, the rate of change is decreasing.

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- What is the practical interpretation of t^* and L?

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- L is the limiting value of the population.

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Year, t	1972	1975	1980	1985	1990	1995	2000	2005
1000s of abortions, A	587	1034	1554	1589	1609	1359	1313	1206

- Table show the number of abortions per year.

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- Table show the number of abortions per year.
- (a) Calculate the average rate of change for the time interval shown between 1972 an 2005.
- (b) What can you say about the sign of d^2A/dt^2 during the period 1972-1995?