Using the fundamental theorem to find the definite integrals

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Fundamental theorem of calculus

Theorem

If $F'(t)$ is continuous for $a \leq t \leq b$, then

$$\int_a^b F'(t)dt = F(b) - F(a)$$

In other words: The definite integral of the derivative of a function gives the total change in the function.

Example

$$\int_1^3 2x dx = ?$$

Example

$$\int_1^3 2x dx = 8.000$$

Example

$$\int_a^b F'(x)dx = F(b) - F(a)$$

- Let $F'(x) = 2x$.

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- Let $F'(x) = 2x$.
- $F(x)$ is an antiderivative of $2x$, take $F(x) = x^2$.
- $\int_1^3 2xdx = \int_1^3 F'(x)dx = F(3) - F(1) = 3^2 - 1^2 = 9 - 1 = 8$

Notation

- $F(x)\Big|_a^b = F(b) - F(a)$

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- $F(x)\Big|_a^b = F(b) - F(a)$
- $\int_1^3 2x dx = x^2\Big|_1^3$.

Example

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- $\int_1^5 (4x + 3) dx = ?$

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- $\int_1^5 (4x + 3) dx = ?$
- $\int_4^9 5\sqrt{x} dx = ?$

Example

Find the area under the graph of $f(x) = 4e^{0.3x}$ between $x = 0$ and $x = 4$

Example

Compute $\int_0^3 2xe^{x^2+1}dx$.

Example

$$\int_1^4 \frac{x^2}{1+x^3}\,dx = ?$$

Method

1. Compute the indefinite integral, expressing an antiderivative in terms of the original variable, and then apply the fundamental theorem.
2. Convert the original limits to new limits in terms of the new variable, do **not** convert the antiderivative back to the original variable, and then apply the fundamental theorem.

Example

$$\int_2^4 \frac{y}{\sqrt{5-y}}dy = ?$$

Example

$$\int_3^5 (\ln t)^2 dt = ?$$