

Fundamental Theorem of Calculus

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Theorem

If $F'(t)$ is continuous for $a \leq t \leq b$, then

$$\int_a^b F'(t) dt = F(b) - F(a)$$

In other words: The definite integral of the derivative of a function gives the total change in the function.

Marginal Cost and Change in Total Cost

Assume $C'(q)$ is the marginal cost and $C(0)$ is the fixed cost.

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- Total cost of producing b units $= C(0) + \int_0^b C'(q) dq$.

Example

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- $2000 + 2059.23 = 4059.23$
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- $\pi'(31) = R'(31) - C'(31)$

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- $\pi'(31) = 200 - \frac{600}{0.3 \cdot 31 + 5} \approx 158.04$