Analyzing Antiderivatives Graphically and Numerically

December 5, 2013

Use Fundamental Theorem to approximate antiderivatives

Suppose $F'(t) = (1.8)^t$ and $F(0) = 2$. Find the value of $F(b)$ for $b = 0, 0.1, 0.2, \ldots, 1.0$.

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- $\int_0^b F'(t)dt + 2 = F(b)$ or $F(b) = \int_0^b F'(t)dt + 2$

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- $\int_0^b F'(t)dt + 2 = F(b)$ or $F(b) = \int_0^b F'(t)dt + 2$
- We can use calculator to find $\int_0^b F'(t)dt$, for $b = 0, 0.1, 0.2, \ldots, 1.0$, and then get $F(b)$.

Use Fundamental Theorem to approximate antiderivatives

$F(0) = 2, \quad F(0.1) = 2.103, \quad F(0.2) = 2.212, \quad F(0.3) = 2.328,$
$F(0.4) = 2.451, \quad F(0.5) = 2.581, \quad F(0.6) = 2.719, \quad F(0.7) = 2.866,$
$F(0.8) = 3.021, \quad F(0.9) = 3.186, \quad F(1.0) = 3.361$

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- $F(b) = \int_0^b F'(t)dt + 2$
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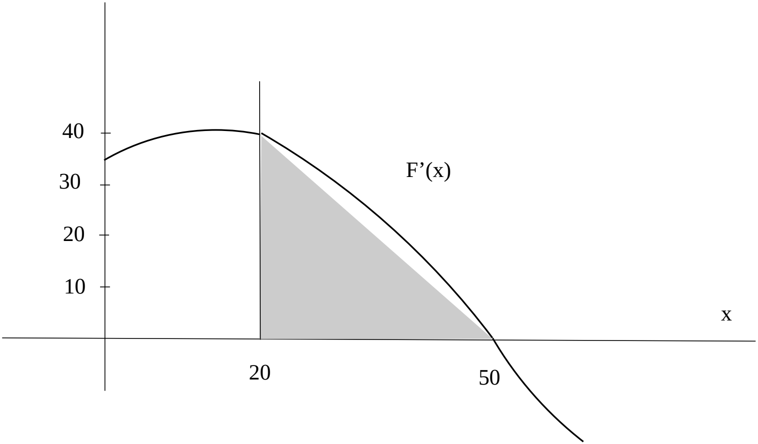
Suppose $F'(t) = (1.8)^t$ and $F(0) = 2$. Find the value of $F(b)$ for $b = 0, 0.1, 0.2, \ldots, 1.0$.

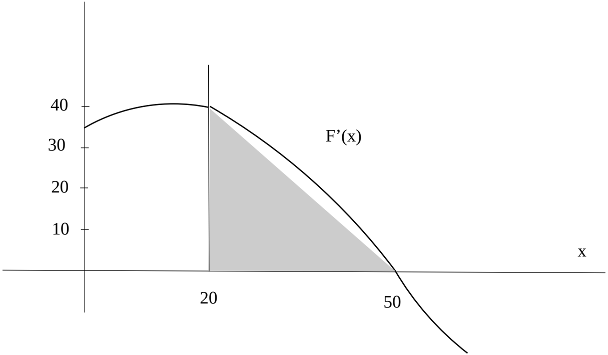
- $F(b) = \int_0^b F'(t)dt + 2$
- We can find find $F(b) = \int_0^b F'(t)dt + 2 = \int_0^b (1.8)^t dt + 2$
- $F(b) = \frac{(1.8)^t}{\ln 1.8}\Big|_0^b + 2 = \frac{(1.8)^b}{\ln 1.8} - \frac{1}{\ln 1.8} + 2$

Example

The graph of the derivative F' of a function F is shown in the figure. Assuming that $F(20) = 40$, estimate the maximum value attained by F.

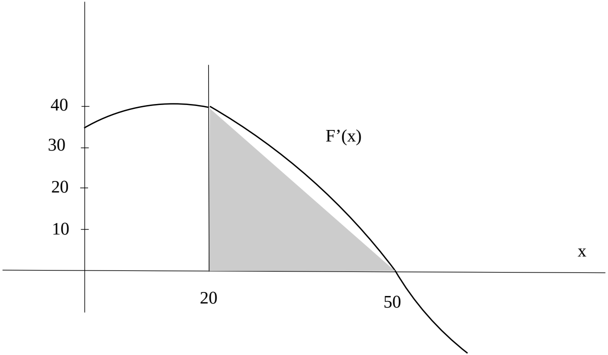


Example



- $F(x)$ is increasing when $x < 50$ and decreasing when $x > 50$.

Example



- $F(x)$ is increasing when $x < 50$ and decreasing when $x > 50$.
- Thus, $F(x)$ get maximum value at $x = 50$, and the maximum value is $F(50)$.

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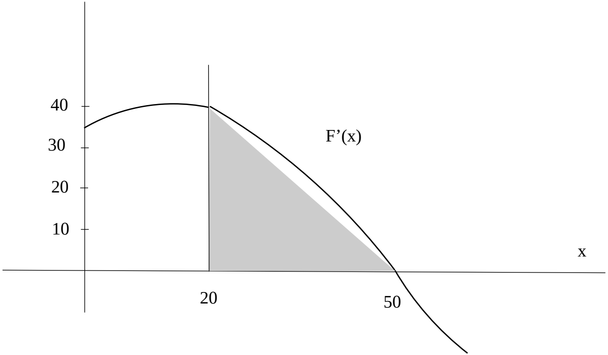

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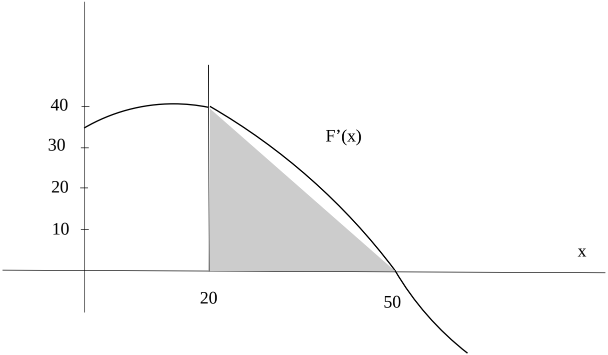
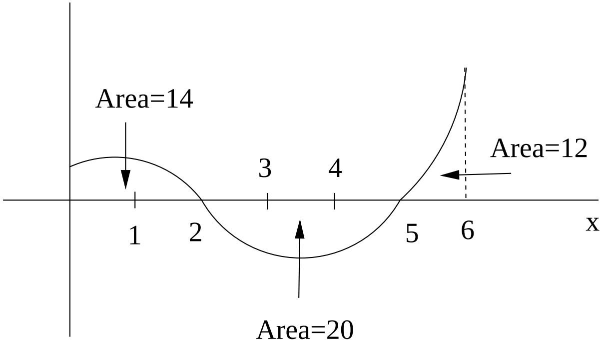


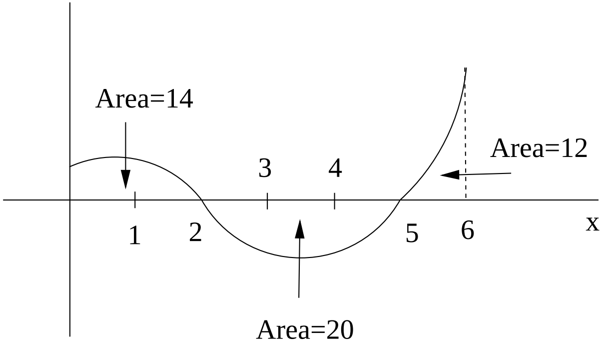
- $F(x)$ is increasing when $x < 50$ and decreasing when $x > 50$.
- Thus, $F(x)$ get maximum value at $x = 50$, and the maximum value is $F(50)$.
- $F(50) - F(20) = \int_{20}^{50} F'(x)dx$
- $F(50) = \int_{20}^{50} F'(x)dx + F(20) = \int_{20}^{50} F'(x)dx + 40$

Example

The graph of $f'(x)$ is shown in the figure. If $f(x) = 10$, sketch a graph of the function f(x).

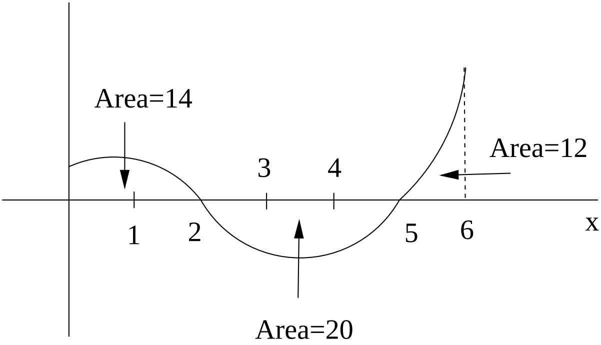


Example



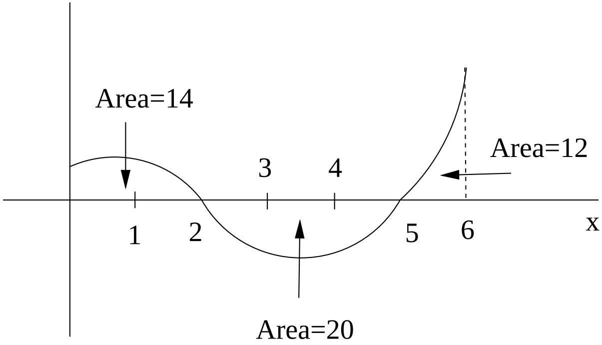
- $f(x)$ is increasing between 0 and 2, decreasing between 2 and 5, and increasing between 5 and 6.

Example



- $f(x)$ is increasing between 0 and 2, decreasing between 2 and 5, and increasing between 5 and 6.
- Local maximum at $x = 2$, local minimum at $x = 5$.

Example



- $f(x)$ is increasing between 0 and 2, decreasing between 2 and 5, and increasing between 5 and 6.
- Local maximum at $x = 2$, local minimum at $x = 5$.
- $f(2) = ?$, $f(5) = ?$, $f(6) = ?$

Example

$f(2) = 14,\ f(5) = 4,\ f(6) = 16.$