

Proportionality and power functions

September 13, 2013

Proportionality

A common functional relationship occurs when one quantity is **proportional** to another. If peaches are \$0.99 a pound, then we say the price you pay, p dollars, is proportional to the weight you buy, q pounds, because

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- This k is called the **constant of proportionality**.
- We also say that y is **inversely proportional** to x if y is proportional to $\frac{1}{x}$.

- The heart mass of a mammal is proportional to its body mass.

Examples

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- Human with a body mass of 70 kg has heart mass of 0.42 kg.
- Estimate the heart mass of horse with a body mass of 650 kg.

Examples

The period of a pendulum, T , is the amount of time required for the pendulum to make one complete swing. For small string, the period, T , is approximately proportional to the square root of l , the pendulum length. So

$$T = k\sqrt{l}$$

.

Examples

An object's weight, w , is inversely proportional to the square of its distance, r , from the earth's center. So

$$w = \frac{k}{r^2}$$

Definition

We say $Q(x)$ is a power function of x if $Q(x)$ is proportional to a constant power of x .

$$Q(x) = k \cdot x^p$$

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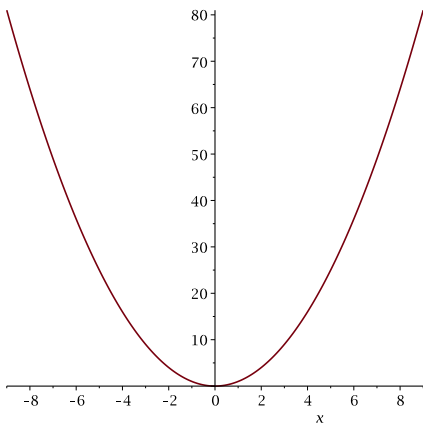
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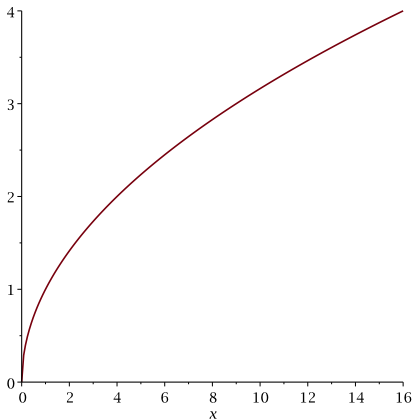
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- $y = (4x^3)^4$.

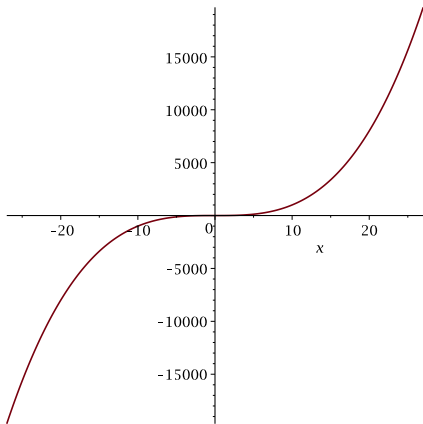
Graph of power functions: $y = x^2$



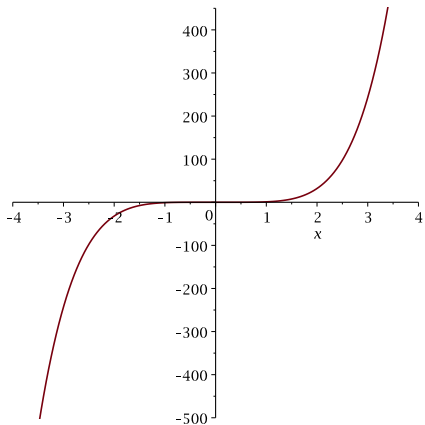
Graph of power functions: $y = \sqrt{x}$



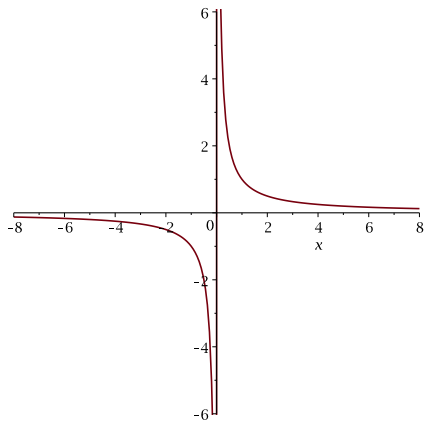
Graph of power functions: $y = x^3$



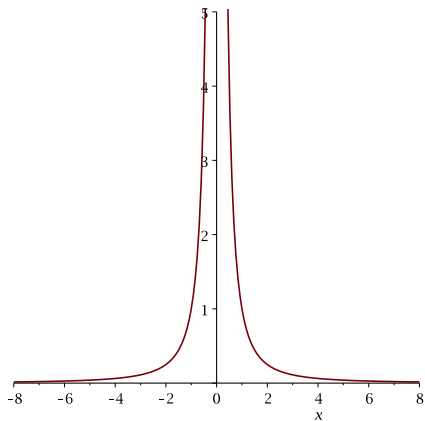
Graph of power functions: $y = x^5$



Graph of power functions: $y = \frac{1}{x}$



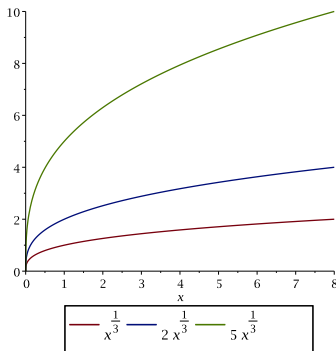
Graph of power functions: $y = \frac{1}{x^2}$



Examples

If N is the average number of species found on an island and A is the area of the island, observations have shown that N is approximately proportional to the cube root of A . So

$$N = k\sqrt[3]{A}.$$



Quadratic functions and Polynomials

Definition

- Sums of power functions with nonnegative integer exponents are called **polynomials**, which are function of the form

$$y = p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0.$$

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- If $n = 2$, the polynomial is called **quadratic function** and has the form $ax^2 + bx + c$ with $a \neq 0$.

Parabola

