Interpretations of The Definite Integral

November 8, 2013

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- Each term of the sums is of the form "$f(x)$ times Δx".
- The unit of measurement for $\int_a^b f(x)dx$ is the product of the units for $f(x)$ and the units for x.

Examples

- If x and $f(x)$ have the same units, then the definite integral $\int_a^b f(x)dx$ is measured in square units, say $cm \times cm = cm^2$.

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- $\int_a^b f(x)dx$ has area-units, if x and $f(x)$ have length-units (e.g. m, cm, km, ft, in...)
- If $f(t)$ is velocity in mph and t is time in hours, then the integral $\int_a^b f(t)dt$ has unit (miles/hour) $\times$ (hour) $=$miles (in fact, the integral represents the total distance traveled).

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- If $f(t)$ is a rate of change, with units of quantity/time, then $f(t)\Delta t$ and the definite integral have units $(quantity/time) \times (time) = quantity$.

Example

A bacteria colony initially has a population of 14 millions bacteria. Suppose that t hours later the population is growing at a rate of $f(t) = 3^t$ million bacteria per hour. Find the population at the times $t = 3$, $t = 4$, and $t = 5$.

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- New population is approximately $14 + 72.82 = 86.82$ millions bacteria.

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- Total change between $t = 0$ and $t = 5$ is $\int_0^5 3^t dt$
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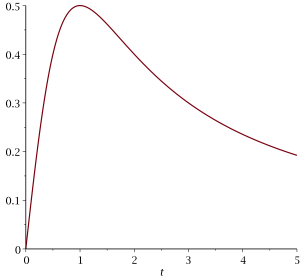
- Total change between $t = 0$ and $t = 5$ is $\int_0^5 3^t dt$
- $\int_0^5 3^t dt \approx 220.28$ millions bacteria.
- New population is approximately $14 + 220.28 = 234.28$ millions bacteria.

Example

After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by

$$r(t) = \frac{t}{t^2+1} \textit{ thousands of antibiodies per minute},$$

where time, t, is in minutes. Assuming that there are no antibodies present at time $t = 0$, find the total antibodies in the blood at the end of 5 minutes.



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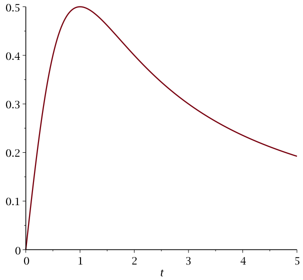

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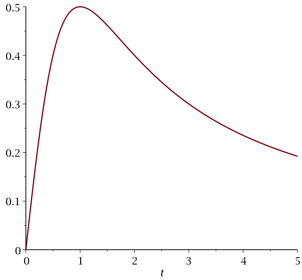

Example

Suppose $C(t)$ represents the cost per day to heat your home in dollars per day, where t is time measured in days and $t = 0$ corresponds to Jan 1. 2010. Interpret $\int_0^{90} C(t)dt$.

- The units of $\int_0^{90} C(t)dt$ is $(dollars/day) \times (day) = dollars$.

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- The units of $\int_0^{90} C(t)dt$ is $(dollars/day) \times (day) = dollars$.
- The integral represents the cost in dollars to heat your house for the first 90 days of 2010.
- i.e. for three months: January, February, and March of 2010.

Example

A man starts 50 miles away from home and takes a trip in his car. He moves on a straight line and his home lies on this line. His velocity is given in the figure.

1. When is the man closest to his home? Approximately how far away (from his starting point) is he then?
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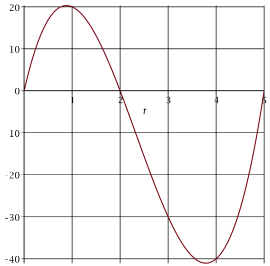

Example

A man starts 50 miles away from home and takes a trip in his car. He moves on a straight line and his home lies on this line. His velocity is given by $V(t) = 5t(t-2)(t-5)$.

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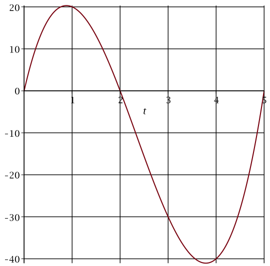
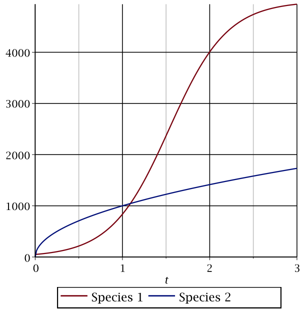


Example

The rate of grow of two populations of two species of plants (measured in new plants per year) are shown in figure. Assume that the population of two species are equal at time $t = 0$.

1. Which population is larger after one year? After two years?
2. How much does the population of species 1 increase during the first two years?



Bio-availability of Drugs

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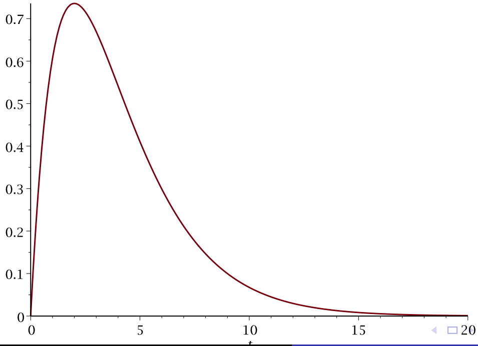
- In pharmacology, the definite integral is used to measure **bio-availability**, that is the overall presence of a drug in the bloodstream during the course of treatment.
- Unit bio-availability represents 1 unit concentration of the drug in the bloodstream for 1 hour.
- For example, a concentration of 3 $\mu g/cm^3$ in the blood for 2 hours has bio-availability of $3 \cdot 2 = 6$ $(\mu g/cm^3) - hours$.

Bio-availability of Drugs

- Ordinarily the concentration of a drug in the blood is not constant.



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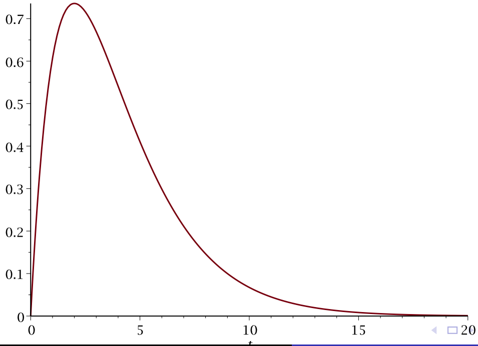
- Typically, the concentration in the blood increases as the drug is absorbed into the bloodstream, and then decreases as the drug is broken down and excreted.



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  $Bio - availablity \ \approx Concentration \times Time = C(t)\Delta t.$
- $Total \ bio - availability \ \approx \sum C(t)\Delta t.$
- Let $\Delta t \to 0$ (i.e., $n \to \infty$),
  $Total \ bio - availability \ = \int_0^T C(t)dt.$

Example

Blood concentration curves of two drugs are given in the figure. Describe the differences and similarities between two drugs in terms of peak concentration, speed of absorbtion into the bloodstream, and total bio-availability.