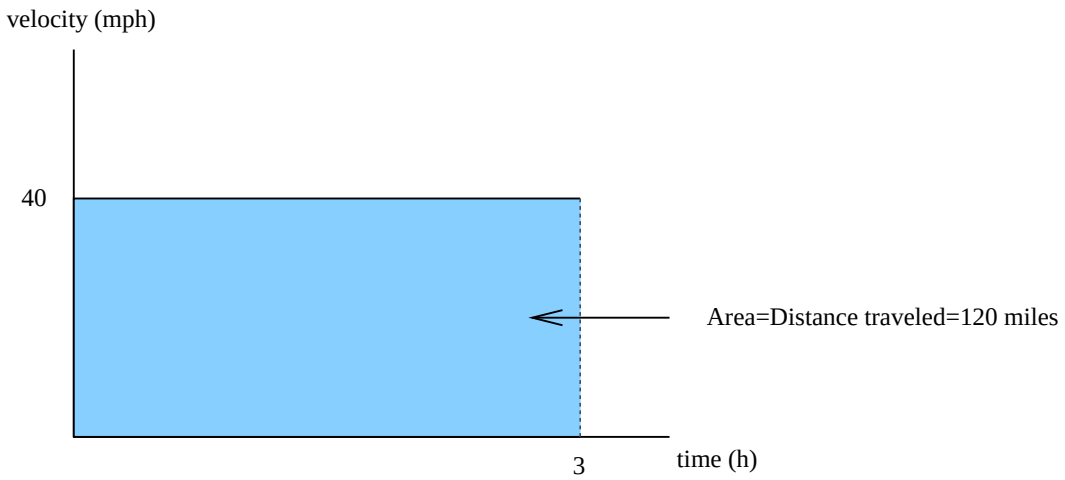
Distance and Accumulated Change

November 1, 2013

How Do We Measure Distance Traveled

- The rate of change of distance with respected to time is velocity.



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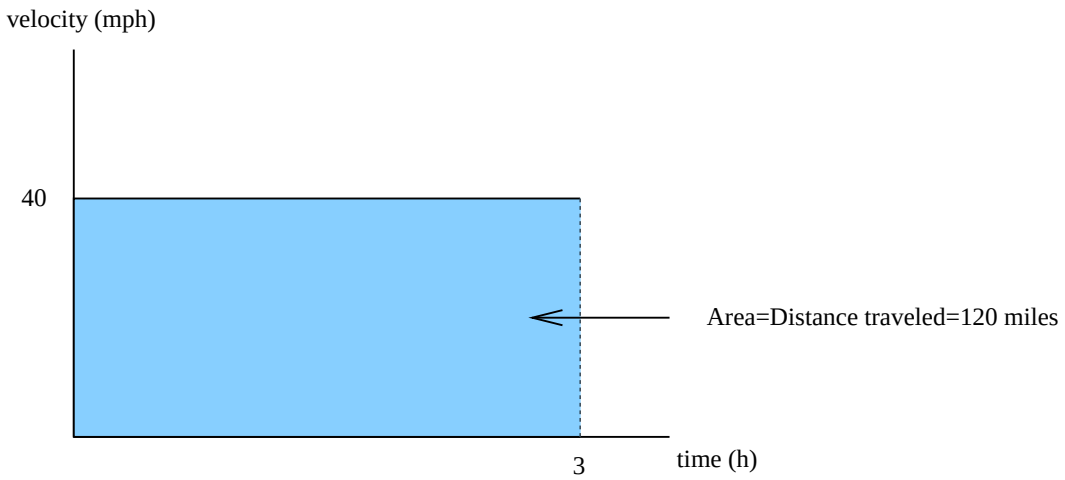

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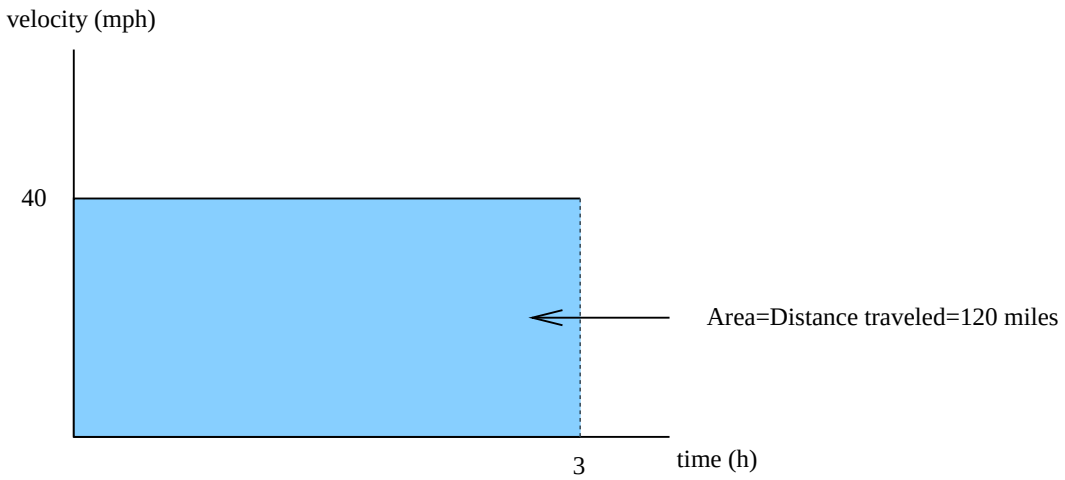

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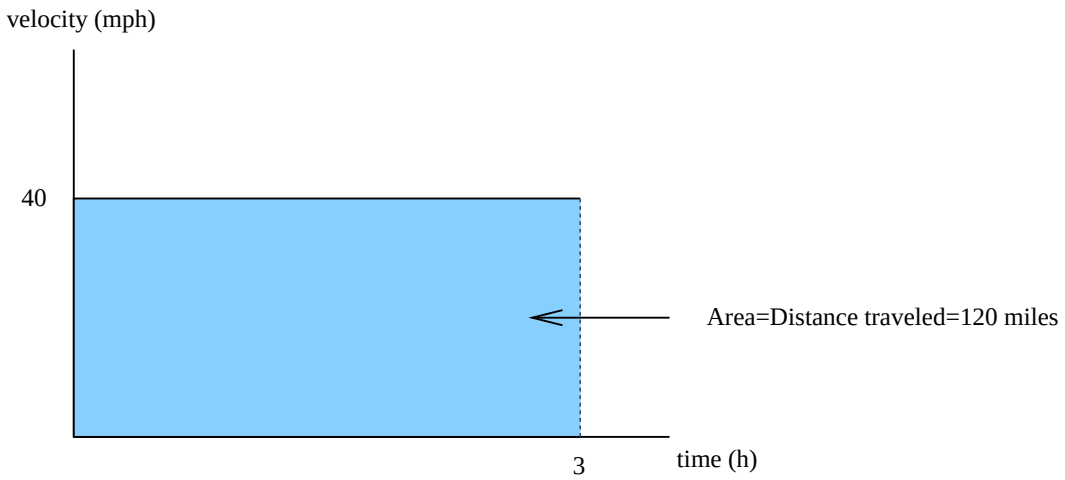
- If we given the velocity, can we find the distance traveled?
- Suppose velocity is constantly 40 mph, what is the distance traveled?
- Distance = Velocity $\times$ Time



Example 1

Suppose that you travel 30mph for 2 hours, then 40 mph for 1/2 hour, then 60 mph for 4 hours. What is the total distance you traveled?

A Thought Experiment: How Far Did the Object Go?

We look at an example where the velocity is continually changing. Suppose an object is moving with increasing velocity and suppose we measure the object's velocity every two seconds. How far has the object traveled?

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	50	86	114	134	146	150

Velocity Data for Every 2 Seconds

- We don't know how fast the object is moving at every moment

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- Since the velocity is increasing, the object is going at least $2 \times 50 = 100$ feet in the first two seconds.

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- Since the velocity is increasing, the object is going at least $2 \times 50 = 100$ feet in the first two seconds.
- Similarly, it goes at least $2 \times 86 = 172$ feet in the next two seconds, and so on

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- Similarly, it goes at least $2 \times 86 = 172$ feet in the next two seconds, and so on
- During the ten-second period it goes at least

$$2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146$$

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$$2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 = 1060$$

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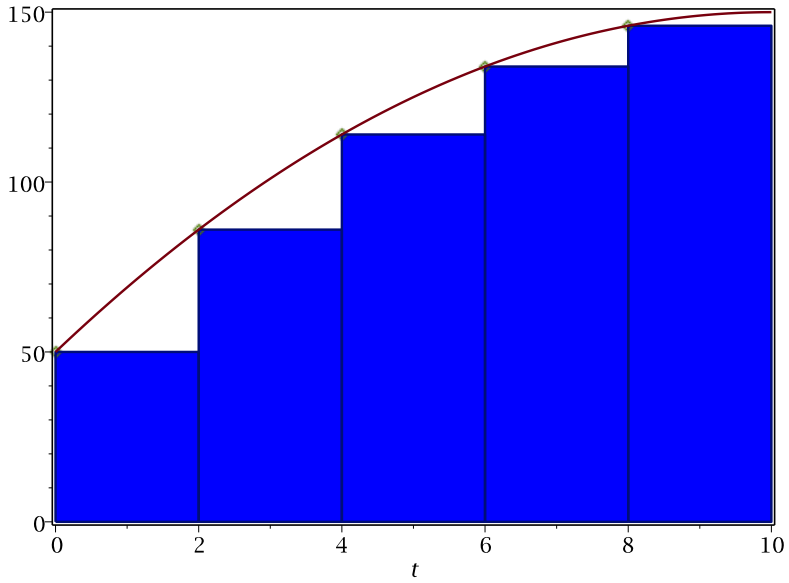
Velocity Data for Every 2 Seconds

Underestimate:

$$2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 = 1060$$



Shaded are estimates distance traveled. Velocity measureed every 2 seconds.

Velocity Data for Every 2 Seconds

- Since the velocity is increasing, the object is going at most $2 \times 86 = 172$ feet in the first two seconds.

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	50	86	114	134	146	150

Velocity Data for Every 2 Seconds

- Since the velocity is increasing, the object is going at most $2 \times 86 = 172$ feet in the first two seconds.
- Similarly, it goes at most $2 \times 114 = 228$ feet in the next two seconds, and so on

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$$2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260$$

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- During the ten-second period it goes at most

$$2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260$$

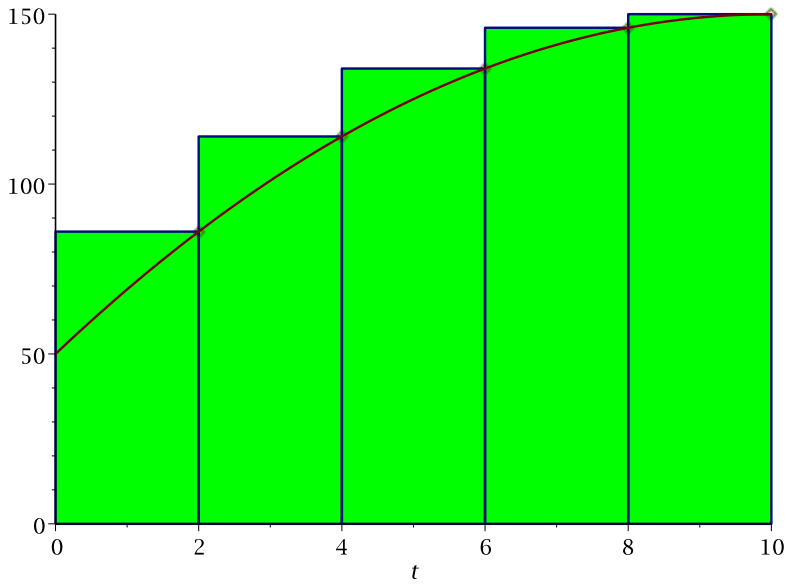
- $$1060 \text{ feet } \leq \text{ Total distance traveled } \leq 1260 \text{ feet}$$

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	50	86	114	134	146	150

Velocity Data for Every 2 Seconds

Overestimate:

$$2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260$$



Shaded are estimates distance traveled. Velocity measureed every 2 seconds.

Velocity Data for Every Second

- Since the velocity is increasing, the object is going at least $1 \times 50 = 50$ feet in the first second.
- Similarly, it goes at least $1 \times 69 = 69$ feet in the next second, and so on
- During the ten-second period it goes at least

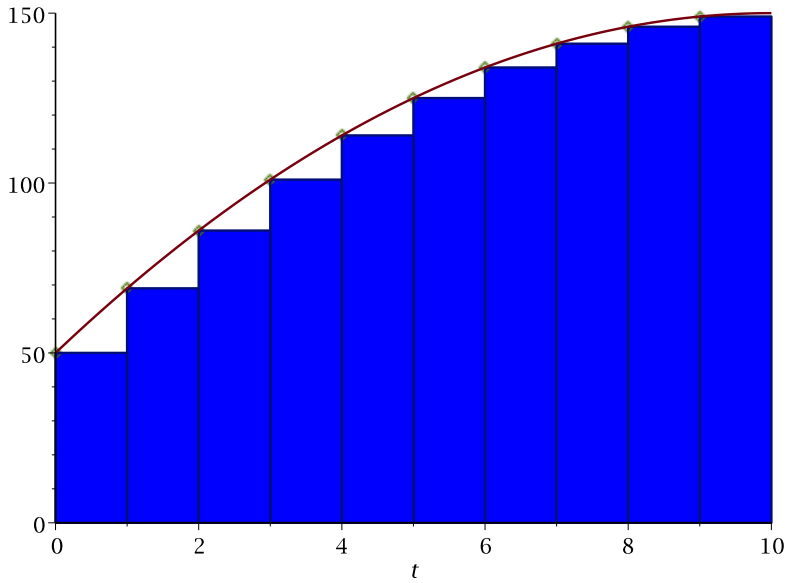
$$1 \times 50 + 1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 +$$
$$+ 1 \times 141 + 1 \times 146 + 1 \times 149 = 1115$$

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (ft/sec)	50	69	86	101	114	125	134	141	146	149	150

Velocity Data for Every Second

Underestimate:

$$1 \times 50 + 1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 +$$
$$+1 \times 141 + 1 \times 146 + 1 \times 149 = 1115$$



Shaded are estimates distance traveled. Velocity measureed every 1 second.

Velocity Data for Every Second

- Since the velocity is increasing, the object is going at most $1 \times 69 = 69$ feet in the first second.
- Similarly, it goes at most $1 \times 86 = 86$ feet in the next second, and so on
- During the ten-second period it goes at most

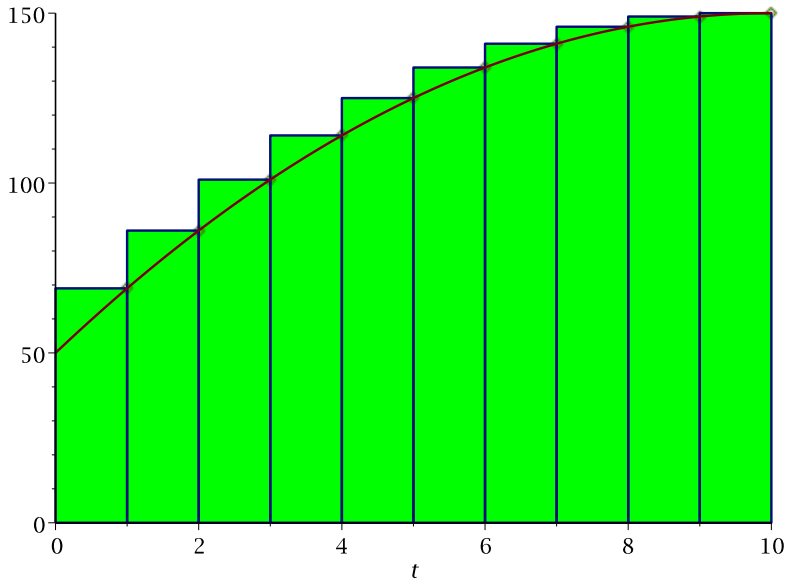
$$1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + 1 \times 141 +$$
$$+ 1 \times 146 + 1 \times 149 + 1 \times 150 = 1215$$

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (ft/sec)	50	69	86	101	114	125	134	141	146	149	150

Velocity Data for Every Second

Overestimate:

$$1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + 1 \times 141 +$$
$$+1 \times 146 + 1 \times 149 + 1 \times 150 = 1215$$



Shaded are estimates distance traveled. Velocity measureed every 1 second.

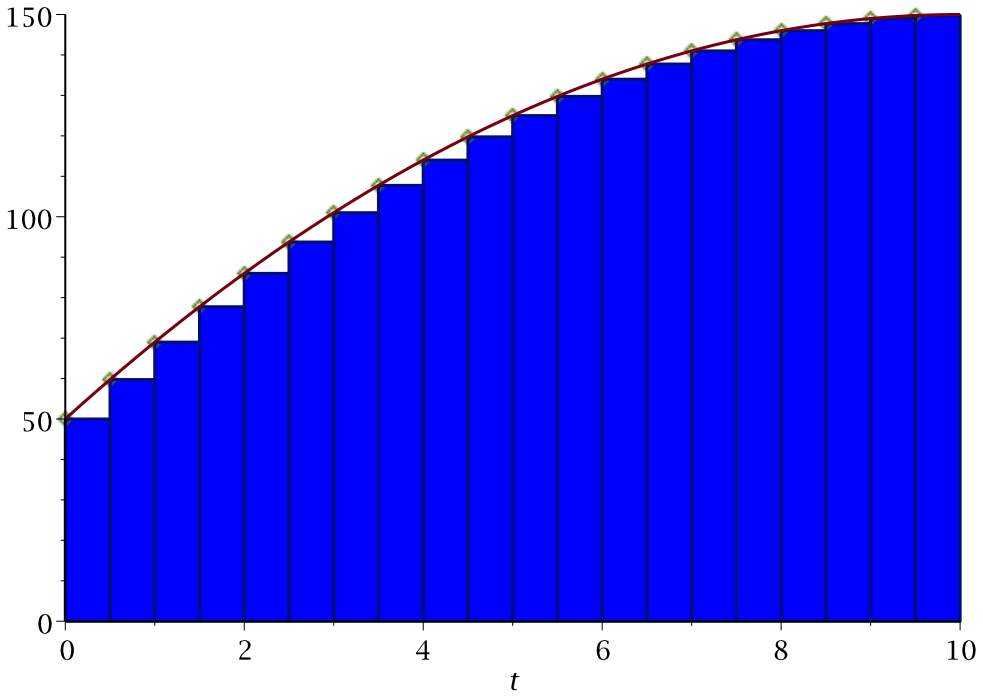
Velocity Data for Every Second

$$1115 \text{ feet } \leq \text{ Total distance traveled } \leq 1215 \text{ feet}$$

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (ft/sec)	50	69	86	101	114	125	134	141	146	149	150

Velocity Data for Every 1/2 Second

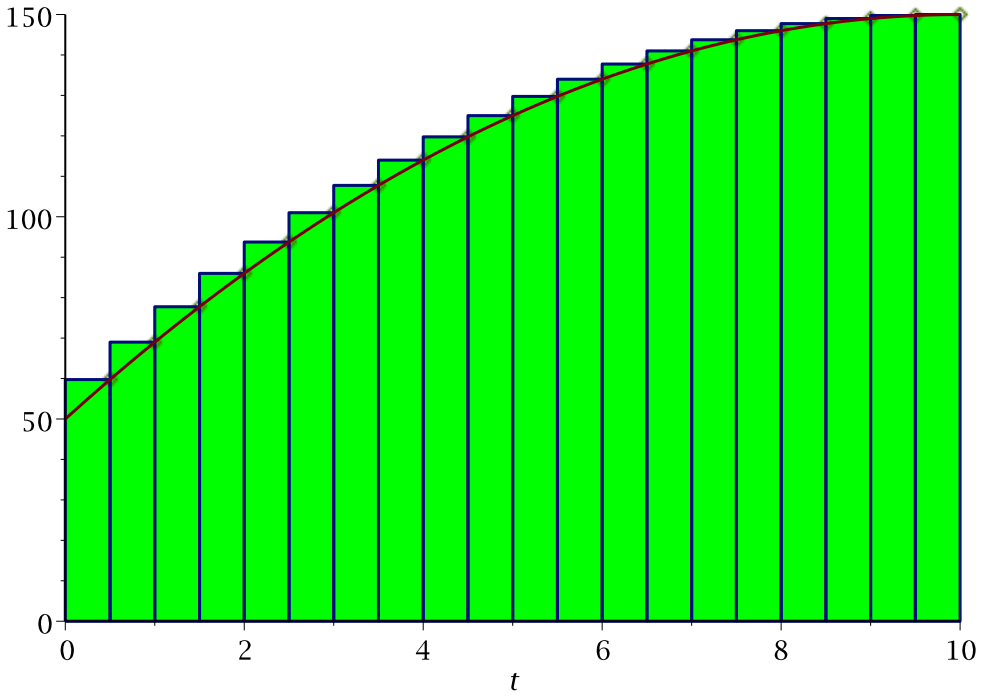
Underestimate



Shaded are estimates distance traveled. Velocity measureed every 1/2 second.

Velocity Data for Every 1/2 Second

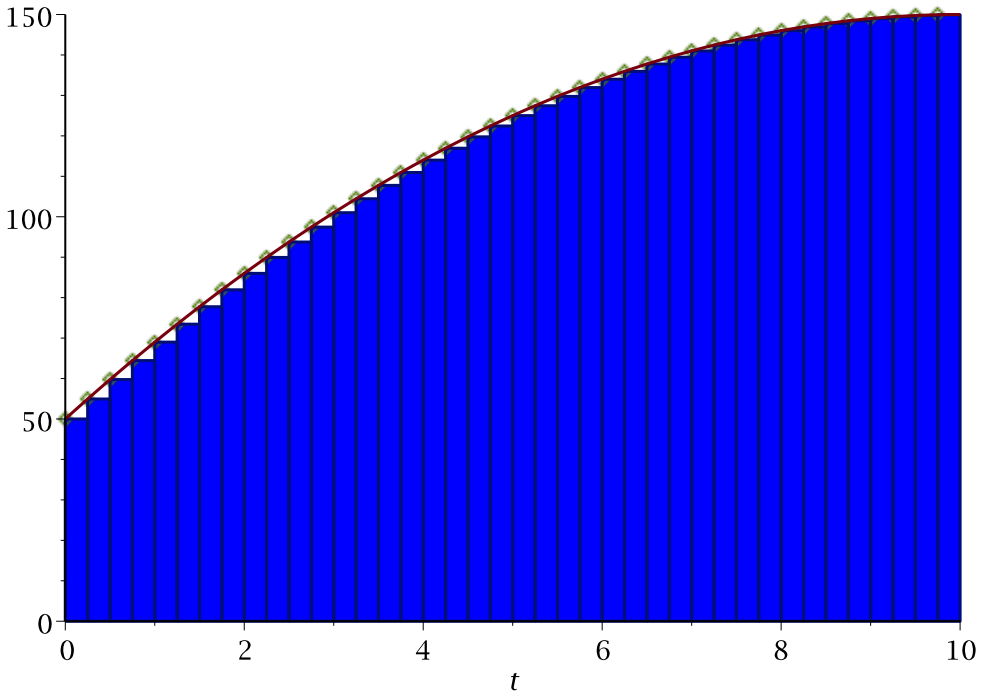
Overestimate



Shaded are estimates distance traveled. Velocity measureed every 1/2 second.

Velocity Data for Every 1/4 Second

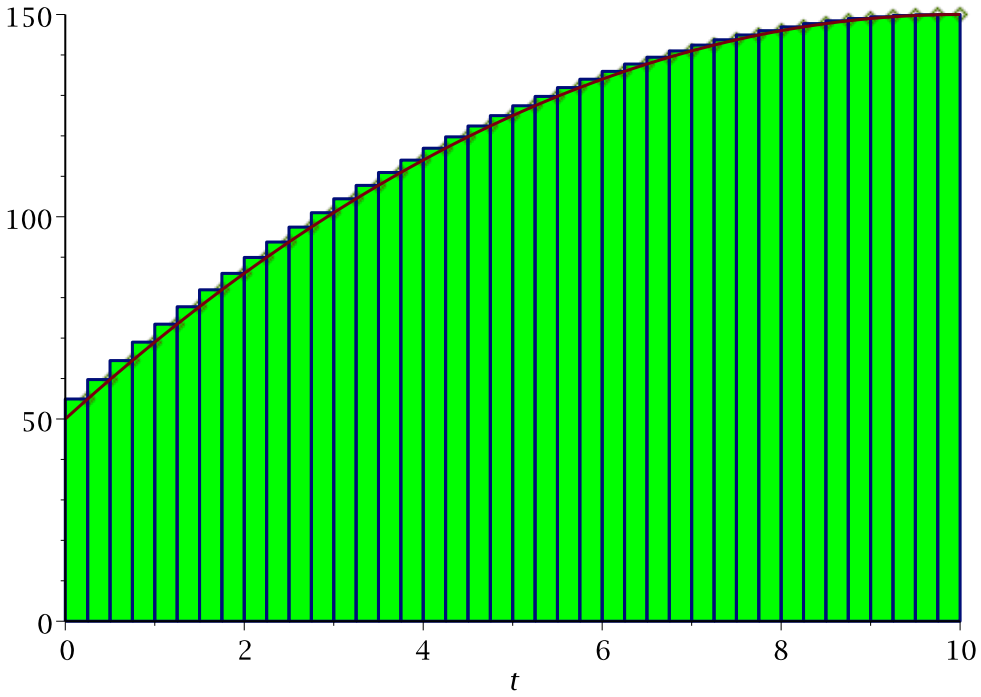
Underestimate



Shaded are estimates distance traveled. Velocity measureed every 1/4 second.

Velocity Data for Every 1/4 Second

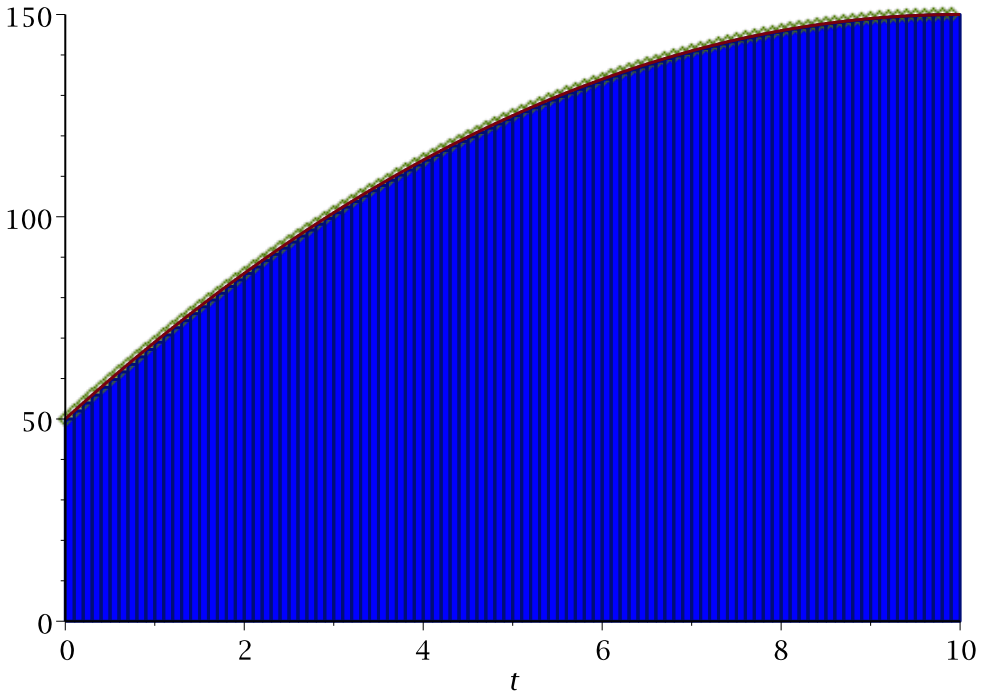
Overestimate



Shaded are estimates distance traveled. Velocity measureed every 1/4 second.

Velocity Data for Every 1/10 Second

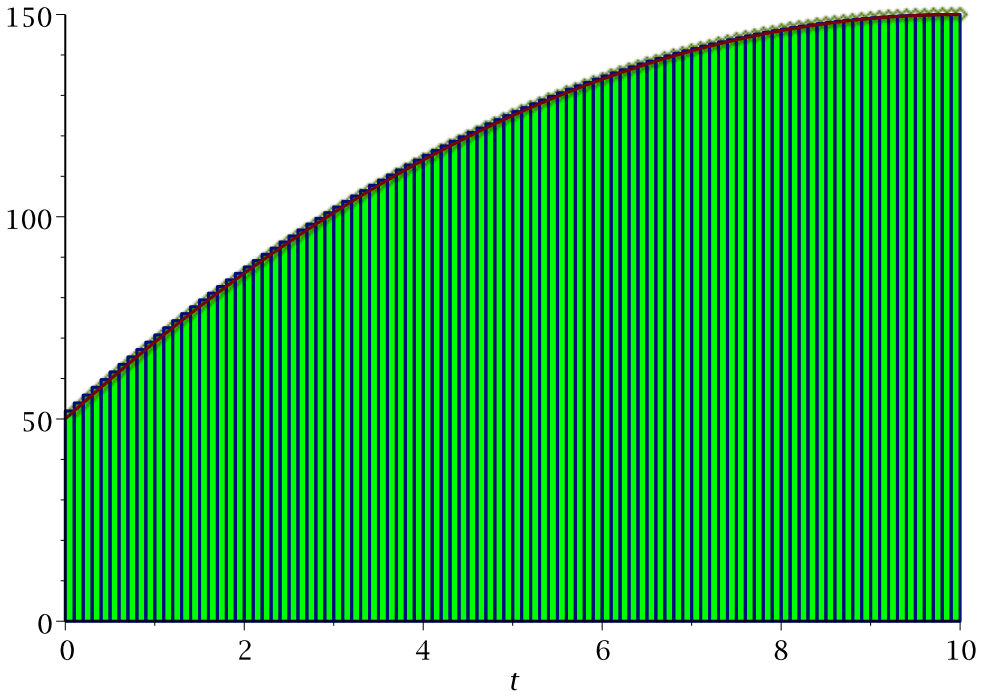
Underestimate



Shaded are estimates distance traveled. Velocity measureed every 1/10 second.

Velocity Data for Every 1/10 Second

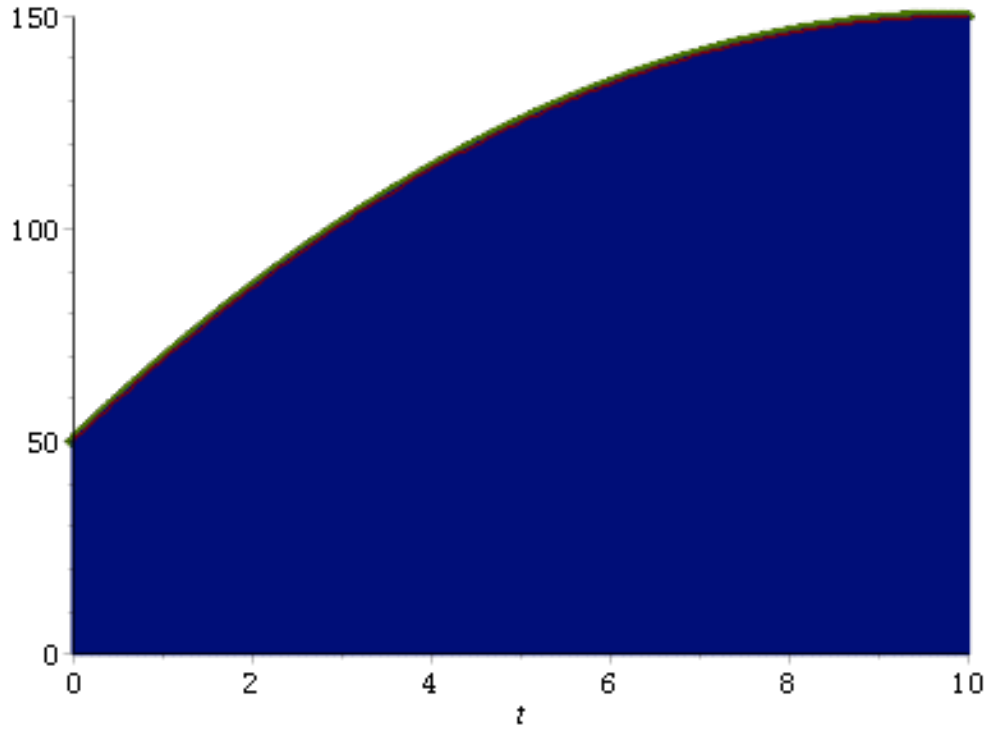
Overestimate



Shaded are estimates distance traveled. Velocity measureed every 1/10 second.

Visualizing Distance on the Velocity Graph: Area Under the Curve

If the velocity is positive, the distance traveled is the area under the velocity curve.



Example

With time t in seconds, the velocity of a bicycle, in feet per second, is given by $v(t) = 4t$ for $0 \leq t \leq 3$, and by $v(t) = 15 - t$ for $t > 3$. How far does the bicycle travel in 7 seconds?

Approximating Total Change from Rate of Change

A city's population grows at the rate of 500 people/year for 3 years and then grows at the rate of 300 people/year for 4 years. What is the total change in the population of the city during this 7-year period?

Example

The rate of sales (in games per week) of a new video game is shown in the table. Assuming that the rate of sales increased throughout the 20-week period, estimate the total number of games sold during this period.

Time (weeks)	0	5	10	15	20
Rate of sales	0	585	892	1875	2350

Example

Total sales= Rate of sales $\times$ Number of weeks.

Time (weeks)	0	5	10	15	20
Rate of sales	0	585	892	1875	2350

Example

The value of a mutual fund increases at a rate of

$$R = 500 \cdot 2^{0.04t},$$

where t is in years since 2010.

1. Using t=0,2,4,6,8,10 makes a table of values for R
2. Use the table to estimate the total change in the value of the mutual fund between 2010 and 2020.