Derivative formulas

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- Example $\frac{d}{dx}(9) = \frac{d}{dx}(3) = 0$.

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- Example $\frac{d}{dx}(4x + 6) = 4$.

Linear properties of derivative

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- $$\frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x)$$

Power rule

For any constant real number n,

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

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- $h(\theta) = \theta^2(\sqrt(\theta) + 4\theta^{-6})$.

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- Given point on the line=?

Example

Find and interpret the second derivative of $f(x) = x^3 + 12x$.

Example

The revenue (in dollars) from producing q units of a product is given by

$$R(q) = 2500q - 5q^2$$

Find and interpret $R(100)$ and $R'(100)$.

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- $y = 5 + 4x^2 + e^{-0.02x}$.
- $P = \sqrt{x} + 3 \cdot 5^x + 4e^{-4x}$

Derivative of $\ln x$

Theorem

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Examples

Find the derivative of $y = 5 \ln x^4 + 6^x + 4e^{-2x}$.

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Suppose \$1000 is deposited into a bank account that pays 8% annual interest, compounded continuously.

- Find $f(10)$ and $f'(10)$.

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Suppose \$1000 is deposited into a bank account that pays 8% annual interest, compounded continuously.

- Find $f(10)$ and $f'(10)$.
- Explain what your answers mean in terms of money.