

Integration by Substitution

November 22, 2013

$$\int 7x^2 \cos(3x^3) dx = ?$$

$$\int 2xe^{x^2+5} dx = ?$$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding **anti-differentiation formula**.

- $f(x) = e^{x^2+4}$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding **anti-differentiation formula**.

- $f(x) = e^{x^2+4}$
- $f(x) = \frac{1}{7}(x^2 + 5)^4$

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding **anti-differentiation formula**.

- $f(x) = e^{x^2+4}$
- $f(x) = \frac{1}{7}(x^2 + 5)^4$
- $f(x) = \ln(x^2 + 1)$

Anti-differentiation formula

An anti-differentiation formula is of the form

$$\int f(x)dx = F(x) + C$$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = e^{x^2+4}$

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = e^{x^2+4}$
- $f'(x) = e^{x^2+4} \cdot 2x$

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = e^{x^2+4}$
- $f'(x) = e^{x^2+4} \cdot 2x$
- $\int f'(x)dx = f(x) + C.$

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = e^{x^2+4}$
- $f'(x) = e^{x^2+4} \cdot 2x$
- $\int f'(x)dx = f(x) + C.$
- $\int e^{x^2+4} \cdot 2xdx = e^{x^2+4} + C.$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = \frac{1}{7}(x^2 + 5)^4$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = \frac{1}{7}(x^2 + 5)^4$
- $\int \frac{4}{7}(x^2 + 5)^3 \cdot 2x dx = \frac{1}{7}(x^2 + 5)^4 + C.$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = \ln(x^2 + 1)$

Chain rule

The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x).$$

Use the chain rule to find $f'(x)$ and then write the corresponding anti-differentiation formula.

- $f(x) = \ln(x^2 + 1)$
- $\int \frac{1}{x^2+1} \cdot 2xdx = \ln(x^2 + 1) + C.$

- The derivative of each inside function is $2x$ in the example. Notice that the derivative of the inside function is a factor in the integrand in each anti-differentiation formula.

- The derivative of each inside function is $2x$ in the example. Notice that the derivative of the inside function is a factor in the integrand in each anti-differentiation formula.
- Finding an inside function whose derivative appears as a factor in the integrand is key to the method of the substitution.

Make a substitution in an integral

- Let w (or v , t ,...) be the inside function.

Make a substitution in an integral

- Let w (or v , t , ...) be the inside function.
- $dw = w'(x)dx$ or $dx = \frac{1}{w'(x)} dw$

Make a substitution in an integral

- Let w (or v , t , ...) be the inside function.
- $dw = w'(x)dx$ or $dx = \frac{1}{w'(x)}dw$
- Express the integrand in terms of w .

Examples

① $\int e^{x^2} \cdot 2x dx = ?$

② $\int (x^2 + 6)^3 \cdot 2x dx = ?$

③ $\int \frac{1}{x^2+6} \cdot 2x dx = ?$

Example

In the previous examples the derivative of the inside function must be present in the integrand for this method to work. Our method works even when the derivative is missing a constant factor.

$$\int 4te^{t^2+3} dt = ?$$

Example

$$\int x^3 \sqrt[3]{x^4 + 7} dx = ?$$

Example

$$\int \frac{x^2}{1+x^3} dx = ?$$

Examples

$$\int \sin(3 - x) dx = ?$$

$$\int e^{2-4x} dx = ?$$

Examples

$$\int 7x^2 \cos(3x^3) dx = ?$$

$$\int 2xe^{x^2+5} dx = ?$$

Example

$$\int e^{\cos \theta} \sin \theta d\theta = ?$$

Examples

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = ?$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$$

Examples

$$\int \frac{e^x}{e^x + 1} dx = ?$$

$$\int \frac{e^x + 1}{\sqrt{e^x + x}} dx = ?$$

Examples

$$\int \sin^n(kx) \cos(kx) dx = ?$$

$$\int \cos^n(kx) \sin(kx) dx = ?$$

Harder Examples

$$\int \frac{1}{1 + e^x} dx = ?$$

$$\int \frac{1}{2 + 3^x} dx = ?$$

$$\int x \ln x dx = ?$$