Global Maxima and Minima

October 24, 2013

Definition

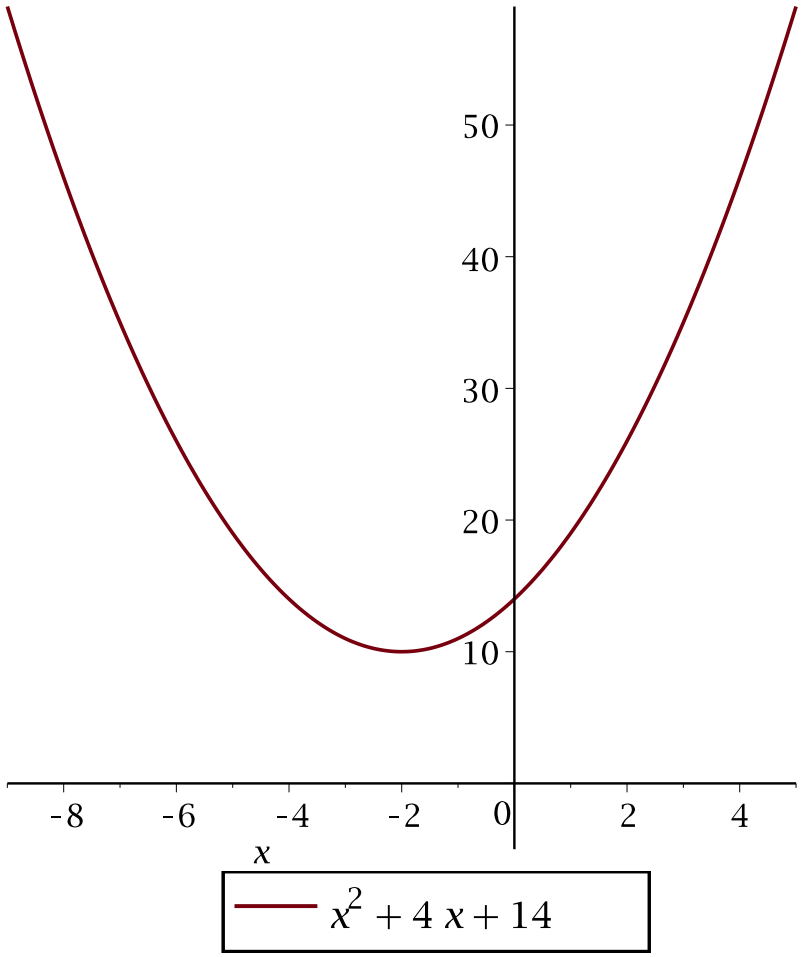
Local maxima and minima occur where a function takes larger or smaller values than at nearby points. However, we a often interested in where a function is larger or smaller than at *all* other points.

Definition

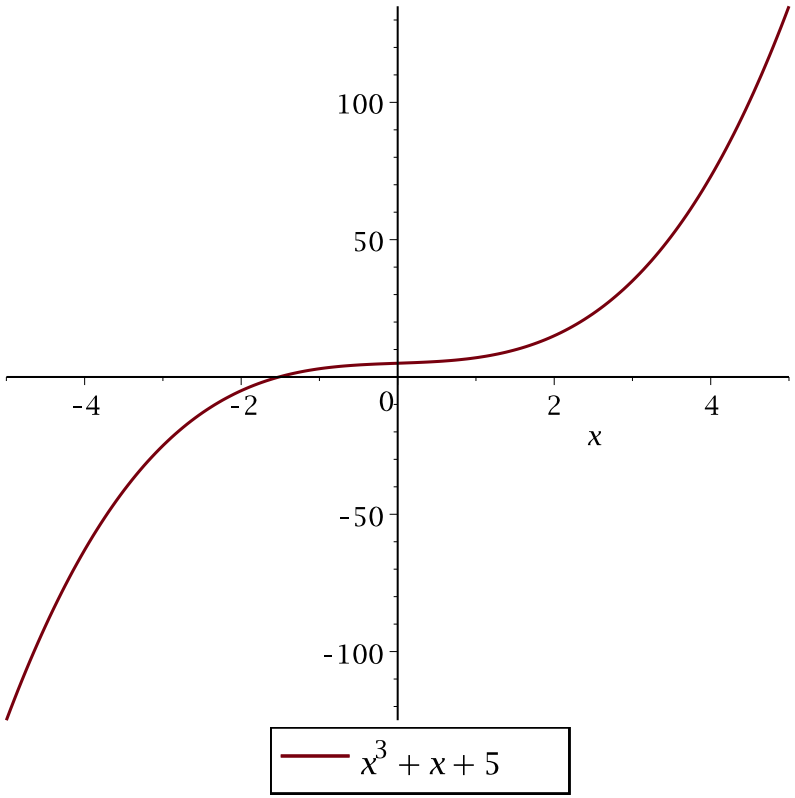
For any function f:

- f has a global minimum at p if $f(p)$ is less than or equal to all values of f.
- f has a global maximum at p if $f(p)$ is greater than or equal to all values of f.

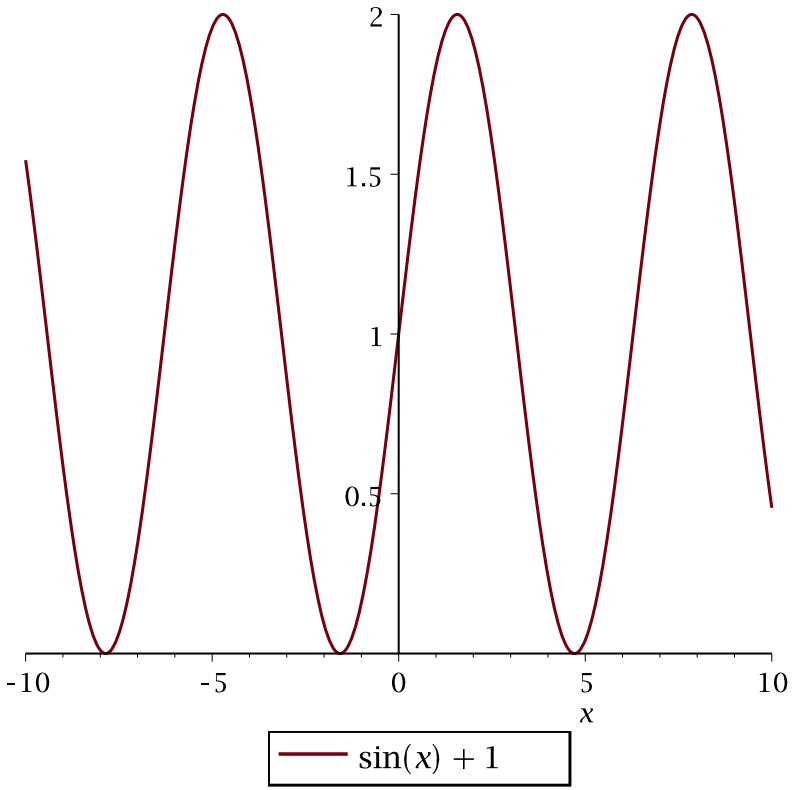
Example



Example

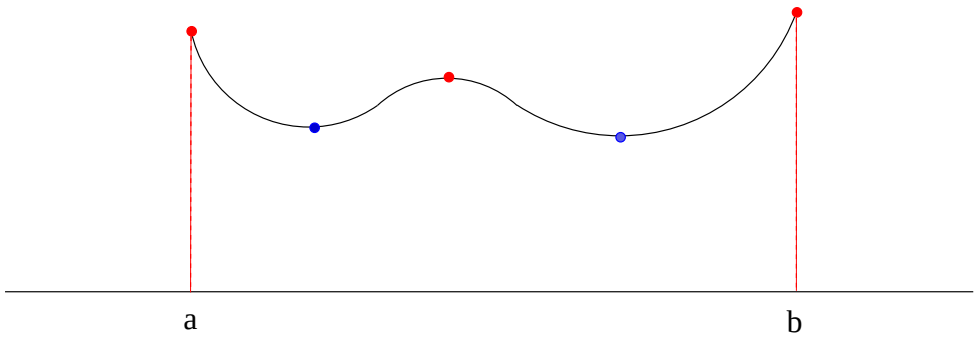


Example



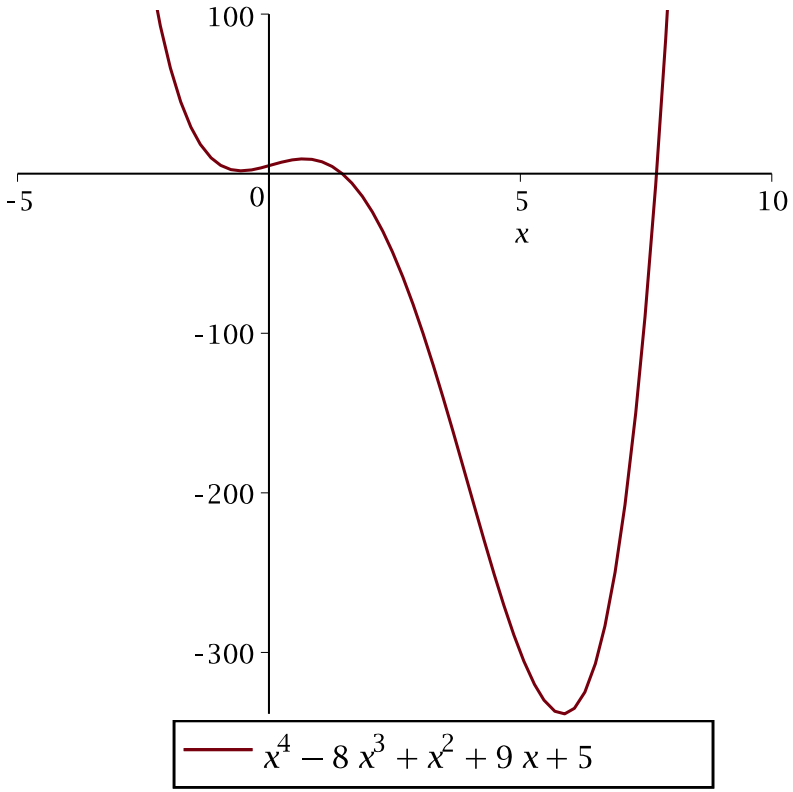
How to find global maxima and minima?

To find the global maximum and minimum of a continuous function on an interval including end points: Compare values of the function at all critical points in the interval and at the endpoints.



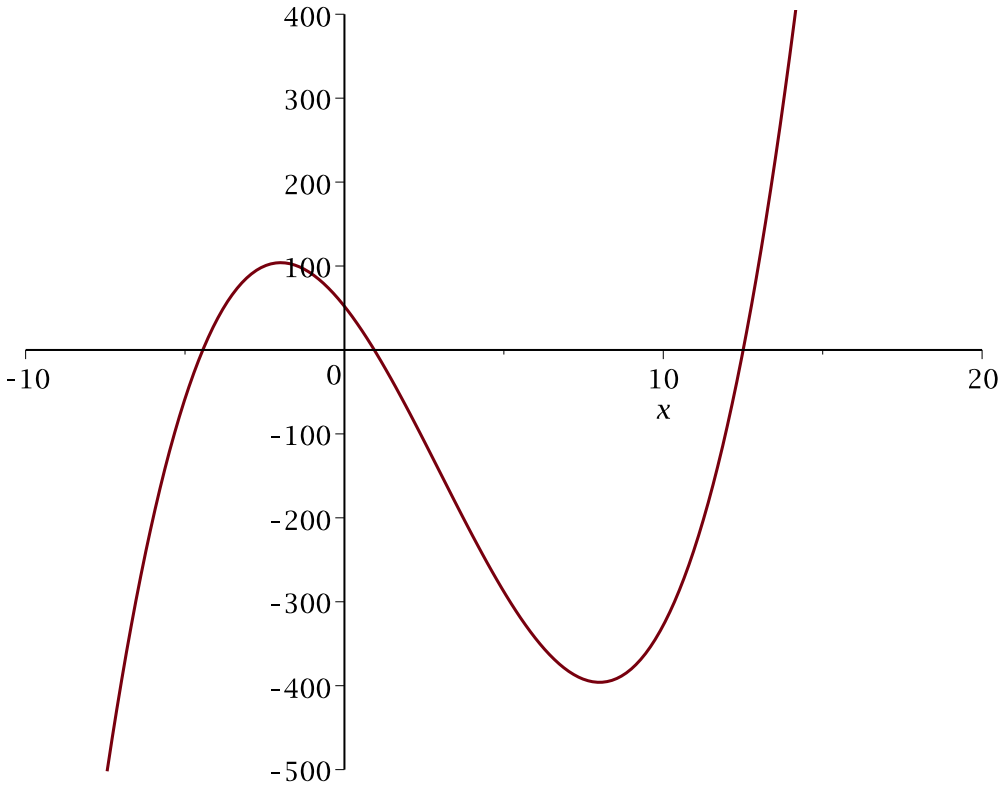
How to find global maxima and minima?

To find the global maximum and minimum of a continuous function on an interval excluding end points or on the entire real line: Find the values of the function at all the critical points and sketch a graph.



Example

$f(x) = x^3 - 9x^2 - 48x + 52.$



Example

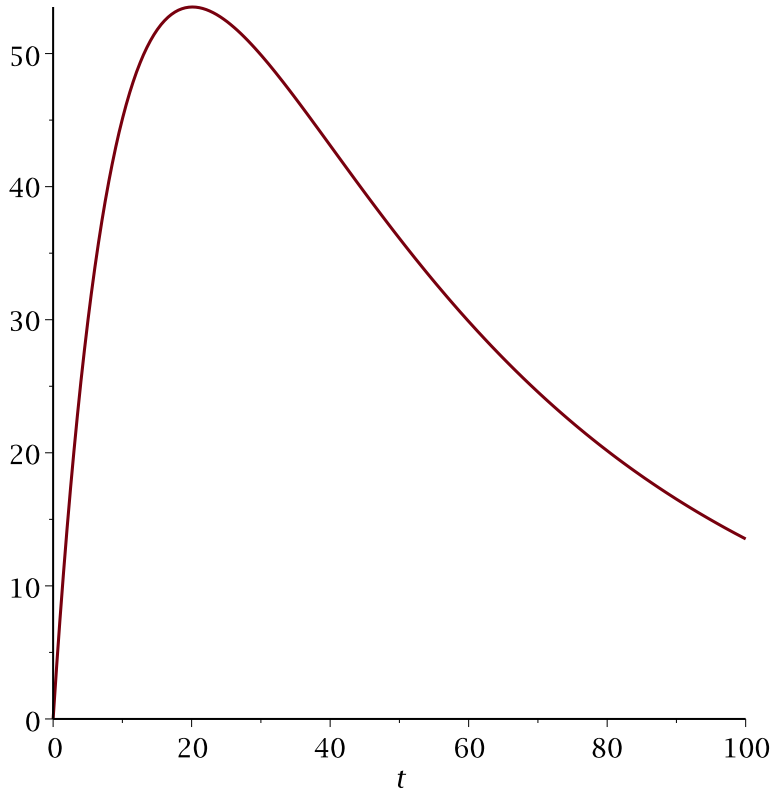
For time , $t \geq 0$, in days, the rate at which photosynthesis takes place in the leaf of a plant, represented by the rate at which oxygen is produced, is approximated by

$$p(t) = 100(e^{-0.02t} - e^{-0.1t}).$$

When is photosynthesis fastest? What is that rate?

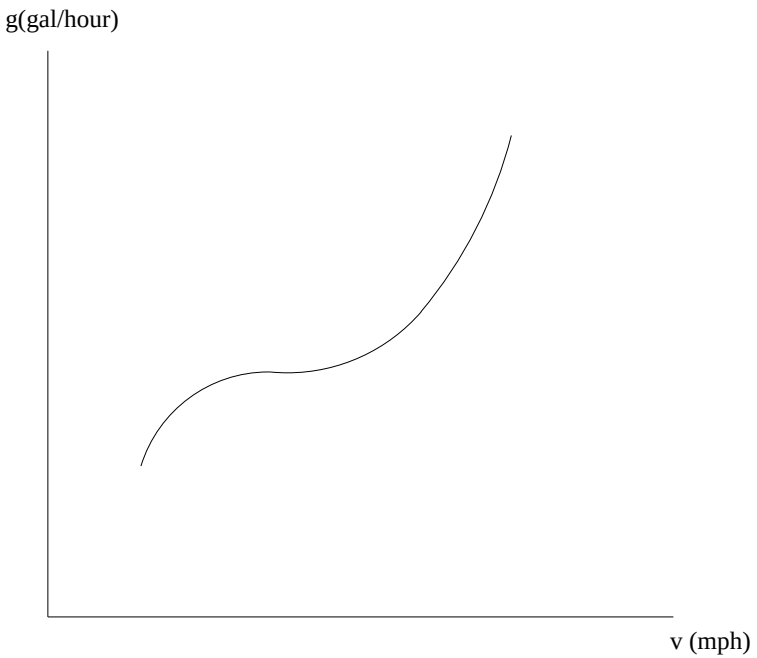
Example

$$p(t) = 100(e^{-0.02t} - e^{-0.1t}).$$



Example: Minimizing Gas Consumption

We investigate is how to set driving speeds to maximize fuel efficiency. We assume that gas consumption, g (in gallons/hour), as a function of velocity, v (in mph) is as shown in figure. We want to minimize the gas consumption per **mile**, not gas consumption per hour. Let $G = g/v$ represent the average gas consumption per mile (The units of G are gallons/mile).



Example: Minimizing Gas Consumption

Minimize $G = g/v$.