Interpretations of the derivative

September 24, 2013

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- Leibniz introduced an alternative notation for derivative.
- For $y = f(x)$

$$f'(x) = \frac{dy}{dx}$$

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- $\dfrac{d}{dx}$ stands for "the derivative with respect to x of ....". Thus dy/dx could be viewed as

$$\frac{d}{dx}(y)$$

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- $$\left.\frac{dy}{dx}\right|_{x=2}$$

Using units to interpret the derivative

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- The units of dA/dB are the unit of A divided by the units of B.
- If the derivative of a function is not changing rapidly near a point, then the derivative is approximately equal to the change in the function when the independent variable increases by 1 unit.

Example

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- We can think dC as the extra cost of building an extra dA square feet.
- If we are planning to build house with area A square feet, $f'(A)$ is approximately the cost per square foot of the extra area involved in building a slightly larger house, and it is call *marginal cost*.

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- So

$$\left.\frac{dC}{dT}\right|_{T=2000} = 100$$

says that when 2000 tons of ore have already been extracted from the mine, the cost of extracting the next ton, the 20001^{st} ton, is about 100\$.

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- *thousand of tons per dollar*
- When the price is \$900, the quantity produced increase by 0.2 thousand tons for one-dollar increase in price.

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- (b) Interpret the statement $f'(20) = 0.2$.

Example

- If the velocity of a car at time t seconds is measure in meters/sec, what is the units of the acceleration?

Using the derivative to estimate values of a function

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- (a) Interpret the statement $f(20) = 11,500$ and $f'(20) = 350$.
- (b) Use part (a) to estimate $f(21)$ and $f(30)$.
- (c) Which estimation in part (b) is more reliable?

Local Linear Approximation

If $y = f(x)$ and Δx is near 0, then $\Delta y \approx f'(x)\Delta x$. Then for x near a and $\Delta x = x - a$,

$$f(x) \approx f(a) + f'(a)\Delta x.$$

This is called the *Tangent Line Approximation*.

(Instantaneous) Relative Rate of Change

Definition

The **(instantaneous) relative rate of change** of $y = f(t)$ at $t = a$ id defined to be

$$\frac{dy/dt}{y} = \frac{f'(a)}{f(a)}.$$

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Example

- Solar photovoltaic (PV) cells are the world's fastest growing energy source. Annual production of PV cells, S, in megawatts, is approximately $S = 277 \exp 0.368t$, where t is in years since 2000. Estimate the relative rate of change of PV cell production in 2020 using
- (a) $\Delta t = 1$,
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- (a) $\Delta t = 0.01$,