

Inflection point

October 23, 2013

Concavity and Inflection Points

A study of the points on the graph of a function where the slope changes the sign led us to critical points. Now we will study the points on the graph where the concavity changes.

Definition

A point at which the graph of a function f changes concavity is called an **inflection point** of f .

The word “inflection point of f ” can refer either to a point in the domain of f or to a point on the graph of f .

How Do You Locate an Inflection Point?

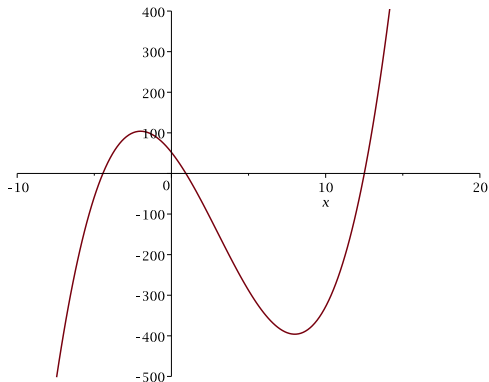
- Concavity of the graph of f changes at an inflection point
- The sign of f'' changes at the inflection point
- At the inflection point, f'' is zero or undefined

Example

Find the inflection point of $f(x) = x^3 - 9x^2 - 48x + 52$.

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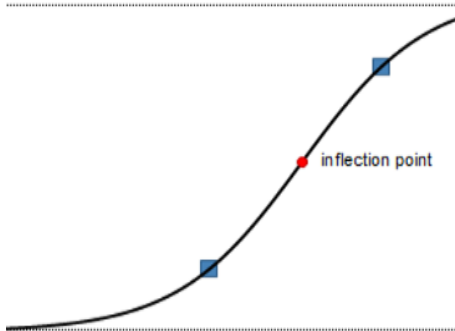
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- f has a critical point at $x = 4$ and an inflection point at $x = 8$;
- the values of f' is negative to the left of 4 and positive to the right of 4;
- the value of f'' is positive to the left of 8 and negative to the right of 8.

Example

Figure shows the population growing toward a limiting population L . There is an inflection point on the graph at the point where the population reaches $L/2$. What is the significance of the inflection point to the population?



Example

How many inflection points and how many critical points does the function $f(x) = x^2 e^{-x}$ have?

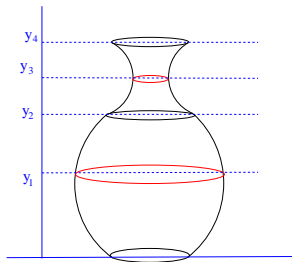
Example

If p is an inflection point of the function f , then $f''(p) = 0$ or undefined. However, the fact $f''(p)$ does NOT imply that p is an inflection point of f . Example: $f(x) = x^4$.

Example

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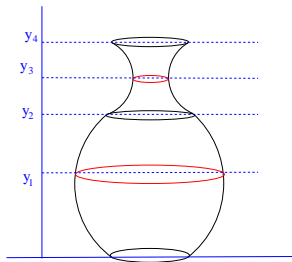
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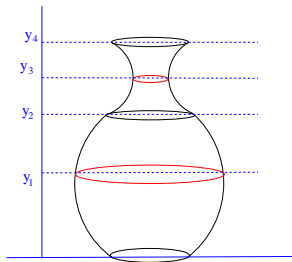
- (a) Graph roughly the function $y = f(t)$, the depth of the water, against time, t .
- (b) Show the inflection point in the graph



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- (a) Graph roughly the function $y = f(t)$, the depth of the water, against time, t .
- (b) Show the inflection point in the graph
- (c) What which depth is $y = f(t)$ growing most quickly?
Most slowly?



Example

What is the concavity of the graph $f(x) = ax^2 + bx + c$?