

Tri Lai

Curriculum Vitae

Department of Mathematics
219 Avery Hall, Lincoln, NE 68588-0130 USA

☎ (402) 472 3731

FAX (402) 472 8466

✉ tlai3@unl.edu

🌐 <http://www.math.unl.edu/~tlai3>

Education

2008–2014 **Ph.D. Mathematics**, Indiana University, Bloomington, IN.

Thesis *Enumeration of tilings of quasi-hexagons, hexagonal dungeons, quartered hexagons, and variants of the Aztec diamond.*

Minor Probability

Advisor Mihai Ciucu

2001–2005 **B.S. Mathematics**, Hanoi University of Science, Hanoi, Vietnam.

Academic Appointments

2016–present **Assistant Professor**, University of Nebraska – Lincoln, Lincoln, NE.

2014–2016 **Postdoctoral Associate**, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN.

2008–2014 **Associate Instructor**, Indiana University, Bloomington, IN.

2005–2008 **Lecturer**, Hanoi National University of Education, Vietnam.

2005–2007 **Lecturer**, FPT University, Vietnam.

Research Interests

Algebraic and Enumerative Combinatorics, Graph Theory, and Cluster Algebras.

Papers published or accepted in referred journals

- 1) **New Aspects of Regions whose Tilings are Enumerated by Perfect Powers**, *Electronic Journal of Combinatorics*, Volume 20, Issue 4 (2013), #P31 (47 pages).
- 2) **Enumeration of Hybrid Domino-Lozenge Tilings**, *Journal of Combinatorial Theory, Series A*, Volume 122 (2014), pp. 53–81.
- 3) **A Proof of Blum’s Conjecture on Hexagonal Dungeons** (with Mihai Ciucu), *Journal of Combinatorial Theory, Series A*, Volume 125 (2014), pp. 273–305.
- 4) **A Simple Proof for the Number of Tilings of Quartered Aztec Diamond**, *Electronic Journal of Combinatorics*, Volume 21, Issue 1 (2014), #P1.6 (13 pages).

- 5) **A Generalization of Aztec Diamond Theorem, Part I**, *Electronic Journal of Combinatorics*, Volume 21, Issue 1 (2014), #P1.51 (19 pages).
- 6) **Enumeration of Tilings of Quartered Aztec Rectangles**, *Electronic Journal of Combinatorics*, Volume 21, Issue 4 (2014), #P4.46 (28 pages).
- 7) **A New Proof for the Number of Lozenge Tilings of Quartered Hexagons**, *Discrete Mathematics*, Volume 338, Issue 11 (2015), pp. 1866–1872.
- 8) **Generating Function of the Tilings of an Aztec Rectangle with Holes**, *Graphs and Combinatorics*, Volume 32, Issue 3 (2016), pp. 1039–1054.
- 9) **A Generalization of Aztec Diamond Theorem, Part II**, *Discrete Mathematics*, Volume 339, Issue 3 (2016), pp. 1172–1179.
- 10) **Double Aztec Rectangles**, *Advances in Applied Mathematics*, Volume 75 (2016), pp. 1–17.
- 11) **A Generalization of Aztec Dragons**, *Graphs and Combinatorics*, Volume 32, Issue 5 (2016), pp. 1979–1999.
- 12) **Majority Digraphs** (with Larry Moss and Jörg Endrullis), *Proceeding of the AMS*, Volume 144, Number 9 (2016), pp. 3701–3715.
- 13) **Enumeration of Antisymmetric Monotone Triangles and Domino Tilings of Quartered Aztec Rectangles**, *Discrete Mathematics*, Volume 339, Issue 5 (2016), pp. 1512–1518.
- 14) **Enumeration of Hybrid Domino-Lozenge Tilings II: Quasi-octagonal Regions**, *Electronic Journal of Combinatorics*, Volume 23, Issue 2 (2016), #P2.2 (25 pages).
- 15) **A q -enumeration of Lozenge Tilings of a Hexagon with Three Dents**, *Advances in Applied Mathematics*, Volume 82 (2017), pp. 23–57.
- 16) **Proof of a Refinement of Blum’s Conjecture on Hexagonal Dungeons**, *Discrete Mathematics*, Volume 340, Issue 7 (2017), pp. 1617–1632.
- 17) **A q -enumeration of Lozenge Tilings of a Hexagon with Four Adjacent Triangles Removed From the Boundary**, *European Journal of Combinatorics*, Volume 64 (2017), pp. 66–87.
- 18) **Beyond Aztec Castles: Toric Cascades in the dP_3 Quiver** (with Gregg Musiker), *Communications in Mathematical Physics*, Volume 356, Issue 3 (2017), pp. 823–881.
- 19) **Perfect Matchings of Trimmed Aztec Rectangles**, *Electronic Journal of Combinatorics*, Volume 24, Issue 4 (2017), #P4.19 (37 pages).
- 20) **Cyclically Symmetric Tilings of a Hexagon with Four Holes** (with Ranjan Rohatgi), *Advances in Applied Mathematics*, Volume 96 (2018), pp. 249–285.
- 21) **Lozenge Tilings of a Halved Hexagon with an Array of Triangles Removed from the Boundary**, *SIAM Journal on Discrete Mathematics*, Volume 32, Number 1 (2018), pp. 783–814.

- 22) **Lozenge Tilings of a Halved Hexagon with an Array of Triangles Removed from the Boundary II**, *Electronic Journal of Combinatorics*, Volume 25, Issue 4 (2018), # P.4.58 (34 pages).
- 23) **Proof a Conjecture of Kenyon and Wilson on Semicontiguous Minors**, *Journal of Combinatorial Theory, Series A*, Volume 161 (2019), pp. 134–163.
- 24) **Enumeration of Lozenge Tilings of a Hexagon with a Shamrock Missing on the Symmetry Axis** (with Ranjan Rohatgi), *Discrete Mathematics*, Volume 342, Issue 2 (2019), pp. 451–472. [The paper has been chosen by the editors of Discrete Mathematics to be included in the Editors' Choice selections for 2019.](#)
- 25) **Lozenge Tilings of Doubly-intruded Hexagons** (with Mihai Ciucu), *Journal of Combinatorial Theory, Series A*, Volume 167 (2019), pp. 294–339.
- 26) **Enumeration of Hybrid Domino-Lozenge Tilings III: Symmetric Tilings**, *Australasian Journal of Combinatorics*, Volume 74, Issue 2 (2019), pp. 253–287.
- 27) **Dungeons and Dragons: Combinatorics for the dP_3 Quiver** (with Gregg Musiker), *Annals of Combinatorics*, Volume 24 (2020), pp. 257–309.
- 28) **Lozenge Tilings of Hexagons with Central Holes and Dents**, *Electronic Journal of Combinatorics*, Volume 27, Issue 1 (2020), #P1.61 (63 pages).
- 29) **Tilings of hexagons with a removed triad of bowties** (with Mihai Ciucu and Ranjan Rohatgi), *Journal of Combinatorial Theory, Series A*, Volume 178 (2021), 105359 (online).
- 30) **Plane partitions of shifted double staircase shape** (with Sam Hopkins) (23 pages), *Journal of Combinatorial Theory, Series A*, Volume 183 (2021), 105486 (online).
- 31) **A Shuffling Theorem for Reflectively Symmetric Tilings**, *Discrete Mathematics*, Volume 344, Issue 7 (2021), 112390 (online).
- 32) **Tiling generating functions of halved hexagons and quartered hexagons** (with Ranjan Rohatgi), *Annals of Combinatorics*, Volume 25 (2021), pp. 471–493.
- 33) **Ratio of tiling generating functions of semi-hexagons and quartered hexagons with dents**, *Enumerative Combinatorics and Applications*, Volume 2, Issue 1 (2022), S2R5.

Preprints

- 34) **A New Proof for a Triple Product Formula for Plane Partitions**, *Submitted for publication* (13 pages). Preprint arxiv: 1710.02241.
- 35) **Tiling Enumeration of Doubly-intruded Halved Hexagons**, *Submitted for publication* (34 pages). Preprint arXiv:1801.00249.
- 36) **Tiling Enumeration of Hexagons with Off-central Holes**, *Submitted for publication* (59 pages). Preprint arXiv:1905.07119.

- 37) **A Shuffling Theorem for Lozenge Tilings of Doubly-Dented Hexagons**, *Submitted for publication* (11 pages) (with Ranjan Rohatgi). Preprint arXiv:1905.08311.
- 38) **A Shuffling Theorem for Centrally Symmetric Tilings**, *Submitted for publication* (26 pages). Preprint arXiv:1906.03759.
- 39) **Tilted Halved Hexagons: Hexagons, Halved hexagons, and Semi-hexagons Under One Roof** (12 pages). Preprint arXiv:2006.10826.
- 40) **Problems in the enumeration of tilings**. Survey paper for the “*Open Problems in Algebraic Combinatorics*” AMS volume to accompany the OPAC 2022 conference at the University of Minnesota. Available online at OPAC 2022’s website and arXiv:2109.01466.

Manuscripts in preparation

- 41) **New Aspects of Lozenge Tilings of Hexagons with a Shamrock Hole** (with Ranjan Rohatgi).
- 42) **Tiling Shuffling Theorems and Characters of Classical Groups**.
- 43) **Beyond Majority Digraphs** (with Larry Moss).
- 44) **Generalize Mills–Robbins–Rumsey’s $(q-)$ Formula for Cyclically Symmetric Plane Partitions**.
- 45) **Generalized Symmetric Shamrock**.
- 46) **Enumeration of Tilings of Holey Hexagonal Dungeon** (Project with grad students).
- 47) **A 3-refinement of Tiling Enumeration of Hexagonal Dungeons** (Project with grad students).
- 48) **Count double-dimer configurations** (Project with grad students).
- 49) **A determinant identity for young tableaux of skew-shapes** (with Igor Pak and Alejandro H. Morales).
- 50) **Double-dimer configurations and quivers of dP_3 (del Pezzo) type** (with Gregg Musiker and Helen Jenne).
- 51-53) **Tilings of Hexagons with intrusions** (a series of papers with Seok Hyun Byun).
- 54) **Generating functions for domino tilings of Ciucu’s cruciform region** (with grad students).
- 55) **Domino tilings of the Aztec triangle** (with Philippe Di Francesco).

Other Publications

- 56) **Proof of a Conjecture of Bauer, Fan and Veldman, 2013** (Undergraduate paper, 25 pp). Preprint: arXiv:1309.5379v2.

Awards, Fellowships and Honors

- 2020 **Discrete Mathematics Editors' Choice 2019** (for the paper "*Enumeration of Lozenge Tilings of a Hexagon with a Shamrock Missing on the Symmetry Axis*" with Ranjan Rohatgi).
- 2018–2023 **Simons Foundation Collaboration Grant #585923**, \$42,000.
- 2014–2016 **Postdoctoral Fellowship** from the Institute for Mathematics and its Applications (IMA), University of Minnesota.
- 2014 **Outstanding Thesis Award** from the Department of Mathematics, Indiana University.
- 2008–2014 **Associate Instructor and Fee Scholarships** from Indiana University.
- 2013 **Glenn Schober Memorial Travel Award** from the Department of Mathematics, Indiana University (April 2013).
- 2012 **William B. Wilcox Mathematics Award** for outstanding graduate student from the Department of Mathematics, Indiana University.
- 2008–2013 **Professional Development Grant** from Vietnam Education Foundation.
- 2008–2010 **Doctoral Fellowship** from Vietnam Education Foundation.
- 2005–2010 **Doctoral Fellowship** from Hanoi University of Science, Vietnam (declined to go to Indiana University).
- 2001 **Encouraged Prize** in Vietnam National Mathematics Olympiad.
- 2000 **Third Prize** in Vietnam National Mathematics Olympiad.

Selected Colloquium, Seminar, and Conference Talks

- May 2022 (Upcoming Conference) **Open Problems in Algebraic Combinatorics**, University of Minnesota, Minneapolis, MN (May 16–20, 2022).
- April 2021 **CombinaTexas 2021 (online)**, hosted by Texas A&M University, College Station, TX, via Zoom (April 9–10, 2021). **Plenary talk:** *Tiling generating functions as classical group characters*.
- August 2020 **Dimers in Combinatorics and Cluster Algebras (online)**, hosted by University of Michigan, Ann Arbor, MI, via Zoom (August 4–10, 2020). Invited talk: *Shuffling Tiling Phenomenon*.
- Feb 2020 **Asymptotic Algebraic Combinatorics**, Institute for Pure and Applied Mathematics (IPAM), University of California, Los Angeles, CA (Feb 4–7, 2020).
- Mar 2019 **CombinaTexas 2019**, Texas A&M University, College Station, TX. Talk title: *Factorization Theorems for Tiling Enumerations of Regions with Holes* (March 23–24, 2019).
- Jan 2019 **Colloquium Lecture**, University of Florida, Gainesville, FL (January 11, 2019). Talk title: *Enumeration of Tilings and Related Problems*.
- Jul 2018 **FPSAC 2018**, Dartmouth College, Hanover, NH (July 16–20, 2018).

- Jun 2018 **Combinatory Analysis 2018 – A Conference in Honor of George Andrews’ 80th Birthday**, Penn State University, State College, PA. Talk title: *Cyclically Symmetric Tilings of Hexagons with Four Holes* (June 21–24, 2018).
- Jun 2018 **SIAM conference on Discrete Mathematics 2018**, Denver, CO. Talk title: *More Duals of MacMahon’s Theorem on Plane Partitions* (June 4–8, 2018).
- Feb 2018 **CombinaTexas 2018**, Texas A&M University, College Station, TX. Talk title: *Tiling Enumeration of Hexagons with Central Holes* (Feb 10–11, 2018).
- Feb 2018 **Geometry Seminar**, Texas A&M University, College Station, TX (Feb 9, 2018). Talk title: *Tilings and More*.
- May 2017 **Midwest Combinatorics Conference**, University of Minnesota, Minneapolis, MN (May 23–25, 2017). Invited talk: *q-Enumerations of Lozenge Tilings*.
- Feb 2017 **Discrete Mathematics Seminar**, University of Nebraska – Lincoln, Lincoln, NE (February 21, 2017). Talk title: *Enumeration of Lozenge Tilings of a Hexagon with Three Dents*.
- Oct 2016 **AMS Central Sectional Meeting**, University of St. Thomas, Minneapolis, MN (October 28–30, 2016). Invited talk: *Enumeration of Domino Tilings of a Double Aztec Rectangle*.
- Sep 2016 **Discrete Mathematics Seminar**, University of Nebraska – Lincoln, Lincoln, NE (September 20, 2016). Talk title: *Proof of a Generalization of the Aztec Diamond Theorem*.
- Feb 2016 **Colloquium Lecture**, University of British Columbia, Vancouver Canada (February 3, 2016). Title: *Tiling Expression of Minors*.
- Feb 2016 **Discrete Mathematics Seminar**, University of British Columbia, Vancouver Canada (February 2, 2016). Invited talk: *Enumeration of Lozenge Tilings of a Hexagon with Three Dents*.
- Jan 2016 **Colloquium Lecture**, University of Nebraska–Lincoln, NE (January 20, 2016). Title: *Tiling Expression of Minors*.
- Nov 2015 **Combinatorics Seminar**, University of California, Los Angeles, CA (November 19, 2015). Invited talk: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.
- Nov 2015 **Integrability and Representation Theory Seminar**, University of Illinois at Urbana-Champaign, IL (November 11, 2015). Invited talk: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.
- Nov 2015 **Combinatorics Seminar**, University of Minnesota, Minneapolis, MN (November 6, 2015). Talk title: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.
- Nov 2015 **Algebra and Combinatorics Seminar**, University of North Carolina State, Raleigh, NC (November 2, 2015). Invited talk: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.

- Oct 2015 **12th Algebra, Geometry and Combinatorics Day (ALGECOM)**, University of Michigan, Ann Arbor, MI (October 24, 2015). Plenary talk: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.
- Oct 2015 **Combinatorics Seminar**, Indiana University, Bloomington, IN (October 6, 2015). Invited talk: *Proof of a Conjecture of Kenyon and Wilson about Semicontiguous Minors*.
- Sep 2015 **IMA Postdoc Show and Tell Seminar**, Minneapolis, MN (September 15, 2015). Talk title: *Enumeration of Tilings and Related Problems*.
- Aug 2015 **8th International Conference on Lattice Path Combinatorics and Applications**, California State Polytechnic University, Pomona, CA (August 16–20, 2015). Invited talk: *Lozenge Tilings of a Hexagon with Three Holes*.
- May 2015 **28th Cumberland Conference on Combinatorics, Graph Theory & Computing**, University of South Carolina, Columbia, SC (May 15–17, 2015). Invited talk: *Lozenge Tilings of Hexagon with Holes and Plane Partitions Fitting in a Special Box*.
- Apr 2015 **AMS Spring Western Sectional Meeting** at University of Nevada, Las Vegas, NV (April 18–19, 2015). Invited talk: *Enumeration of Lozenge Tilings of a Hexagon with Holes on Boundary*.
- Mar 2015 **AMS Central Spring Sectional Meeting** at Michigan State University, East Lansing, MI (March 14–15, 2015). Invited talk: *Lozenge Tilings of a Hexagon with Holes on Boundary and Plane Partitions that Fit in a Special Box*.
- Mar 2015 **Combinatorics Seminar**, Department of Mathematics, Indiana University, Bloomington, IN (March 10, 2015). Invited talk: *Enumeration of Lozenge Tilings of a Hexagon with a Shamrock-shaped Hole on Boundary*.
- Feb 2015 **IMA seminar**, Minneapolis, MN (February 3, 2015). Talk title: *Enumeration of Lozenge Tilings of a Hexagon with Holes on Boundary*.
- Dec 2014 **Combinatorics Seminar**, School of Mathematics, University of Minnesota, Minneapolis, MN (December 5, 2014). Talk title: *Proof of a Generalization of Aztec Diamond Theorem*.
- Oct 2014 **Combinatorics Seminar**, Department of Mathematics, Indiana University, Bloomington, IN (October 7, 2014). Talk title: *Enumeration of Tilings of Quartered Aztec Rectangles*.
- Sep 2014 **AMS Central Fall Section Meeting**, University of Wisconsin–Eau Claire, Eau Claire, WI (September 20–21, 2014). Invited talk: *Proof of Blum’s Conjecture on Hexagonal Dungeons*.
- Sep 2014 **IMA Postdoc Show and Tell Seminar**, Minneapolis, MN (September 16, 2014). Talk title: *Exact Enumeration of Tilings*.
- Oct 2013 **Combinatorics Seminar**, Department of Mathematics, Indiana University, Bloomington, IN (October 14, 2013). Talk title: *Proof of Blum’s Conjecture on Hexagonal Dungeons*.

- Oct 2013 **Combinatorics Seminar**, Department of Mathematics, Indiana University, Bloomington, IN (October 7, 2013). Talk title: *Enumeration of Hybrid Domino-Lozenge Tilings*.
- Apr 2013 **Graduate Student Combinatorics Conference** at The University of Minnesota, Minneapolis, MN (April 19–21, 2013). Invited talk: *Subgraph Replacements in Enumeration of Tilings*.
- Mar 2013 **AMS Spring Southeastern Sectional Meeting**, University of Mississippi, Oxford, MS (March 1–3, 2013). Invited talk: *Enumeration of Hybrid Domino-Lozenge Tilings*.

Teaching Experience

All courses listed below are taught as *instructor of record* with full course responsibilities

2016–present **At University of Nebraska – Lincoln.**

- MATH 314 (Linear Algebra), Fall 2016 (two classes)
- MATH 314 (Linear Algebra), Spring 2017
- MATH 314 (Linear Algebra), Spring 2017
- MATH 221 (Differential Equations), Fall 2017
- MATH 450 (Combinatorics), Fall 2017
- MATH 958 (Topic in Discrete Mathematics), Spring 2018
- MATH 850 (Discrete Mathematics I), Fall 2018
- MATH 314 (Linear Algebra), Fall 2018
- MATH 852 (Discrete Mathematics II), Spring 2019
- MATH 450 (Combinatorics), Fall 2019
- MATH 314 (Linear Algebra), Fall 2019
- MATH 314 (Linear Algebra) (*coordinator*), Spring 2020
- MATH 314 (Linear Algebra) (*coordinator*), Fall 2020
- MATH 310 (Modern Algebra), Fall 2020
- MATH 852 (Discrete Mathematics II), Spring 2021
- MATH 850 (Discrete Mathematics I), Fall 2021
- MATH 309 (Introduction to Mathematical Proofs), Fall 2021
- MATH 852 (Discrete Mathematics II), Spring 2022

2008–2014 **At Indiana University.**

- M119 (Brief Survey of Calculus I), Fall 2013
- M118 (Finite Mathematics), Spring 2013
- M118 (Finite Mathematics), Summer 2012
- M025 (Precalculus), Spring 2012 (two classes)

2005–2007 **At Hanoi National University of Education (Hanoi, Vietnam).** *Discrete Mathematics.*

2006–2008 **At FPT University (Hanoi, Vietnam).**

Discrete Mathematics.

Graduate Students Supervised

2017–2021 Thesis advisor for

- Suji Hong – Now a Lecture at Yale University (*Co-advise with Kyungyong Lee of University of Alabama*).

Postdocs Mentoring

2019–2021 Co-mentor with Jamie Radcliffe and Xavier Pérez–Giménez for

- Abigail Raz (Now an Assistant Professor at The Cooper Union).

Professional Activities

Referee:

- Electronic Journal of Combinatorics
- Graphs and Combinatorics
- Australasian Journal of Combinatorics
- European Journal of Combinatorics
- SIAM Journal on Discrete Mathematics
- Advances in Applied Mathematics
- Discrete Mathematics
- Journal of Integer Sequences
- Journal of Combinatorial Theory, Series A
- Algebraic Combinatorics
- International Mathematics Research Notices
- Transaction of AMS
- Annals of Combinatorics
- Combinatorial Theory
- Enumerative Combinatorics and Applications

Reviewer:

- AMS Mathematical Reviews.
- CRC Press–Chapman Hall (book reviewer).
- French National Research Agency (grant reviewer).

Seminar organizer:

- Algebraic Combinatorics and Mathematical Physics Seminar, University of Nebraska –Lincoln.

Internal service

- 2016–present *Undergraduate Activities Committee (chair):*
- Faculty advisor Pi Mu Epsilon.
 - Faculty advisor for Math Club.
 - Organizer for Undergraduate Course Preview Section.
 - Organizer of Math Club talks.
 - Speaker for Math Club (Jan 2017; Talk title: *Mathematics of Tilings*).
 - Organizer of “Career Perspectives in Math Lecture 2019, 2020”
 - Organizer of “Central State Mathematics Undergraduate Research Conference 2018.”
- Fall 2017 *Grader for the Nebraska Math Day:* a high school mathematics contest hosted by the Department of Mathematics, University of Nebraska–Lincoln.
- Spring 2017 *UNL Graduate Fellowship Committee (fellowship reviewer).*
- 2019–present *RTG proposal preparing Committee.*
- AY 2019– 2020 *Tenure-track Hiring Committee.*
- Spring 2020 *Textbook Reviewing Committee.*
- 2017–present *Ph.D. Supervising Committee Member* for George Nars, Math Department, University of Nebraska –Lincoln.
- 2018–present *Ph.D. Supervising Committee Member* for Jared Soundy, CS Department, University of Nebraska –Lincoln.
- 2019–present *Ph.D. Supervising Committee Member* for Michelle Harver, Math Department, University of Nebraska –Lincoln.
- 2020–2021 *Master’s Supervising Committee Member* for Cuong Than, CS Department, University of Nebraska –Lincoln.

Professional Memberships

- 2008–present American Mathematical Society.

Languages

Vietnamese: Native.

English: Professional working proficiency.