1. Suppose $X$ is a continuous random variable with mean and variance both equal to 20. What can be said about $\mathbb{P}[0 \leq X \leq 40]$?

2. (a) Look up the distribution of a Poisson random variable with parameter $\lambda$, state it and use that to calculate $\mathbb{P}[X \geq 1]$, and $\mathbb{P}[X \geq 2]$ where $X$ is a Poisson random variable with parameter 1.

   (b) Given that the m.g.f. $\phi_X(t)$ of a Poisson random variable with parameter $\lambda$ is $e^{\lambda(e^t-1)}$, show that the sum of independent Poisson random variables $X_1$ with parameter $\lambda_1$ and $X_2$ with parameter $\lambda_2$ is again Poisson with parameter $\lambda_1 + \lambda_2$.

   (c) Using the fact that the sum of two independent Poisson random variables with means $\lambda_1$ and $\lambda_2$ is again Poisson with mean $\lambda_1 + \lambda_2$ find the exact probability that $\mathbb{P}[X_1 + \cdots + X_{10} > 15]$ where each $X_i$ is a Poisson random variable with parameter 1.

   (d) Use the Markov Inequality to get a bound on $\mathbb{P}[X_1 + \cdots + X_{10} > 15]$ where each $X_i$ is a Poisson random variable with parameter 1.

3. A first simple assumption is that the daily change of a company’s stock on the stock market is a random variable with mean 0 and variance $\sigma^2$. That is, if $S_n$ represents the price of the stock on day $n$ with $S_0$ given, then 

$$S_n = S_{n-1} + X_n, n \geq 1$$
where \( X_1, X_2, \ldots \) are independent, identically distributed continuous random variables with mean 0 and variance \( \sigma^2 \). (Note that this is an additive assumption about the change in a stock price. In the binomial tree models, we assumed that a stock’s price changes by a *multiplicative factor* up or down. We will have more to say about these two distinct models later.) Suppose that a stock’s price today is 100. If \( \sigma^2 = 1 \), what can you say about the probability that after 10 days, the stock’s price will be between 95 and 105 on the tenth day?

4. Find the moment generating function \( \phi_X(t) = \mathbb{E} [\exp(tX)] \) of the random variable \( X \) which takes values 1 with probability \( \frac{1}{2} \) and \(-1\) with probability \( \frac{1}{2} \). Show directly (that is, without using Taylor polynomial approximations) that \( \phi_X(t/\sqrt{n})^n \to \exp(t^2/2) \). (Hint: Use L’Hopital’s Theorem to evaluate the limit, after taking logarithms of both sides.)