Math489/889
Stochastic Processes and
Advanced Mathematical Finance
Homework 5

Steve Dunbar
Due Wed, October 6, 2010

1. Use a fair coin, say a penny, to play a simple coin-flipping game, as
described throughout the chapter. Use the chart in the section to record
the outcomes of the game. Save your chart as you will use this random
record several times later in the course to test and illustrate some of
the theorems. Each “gambler” flips the coin, and records a $1 (gains
$1) if the coin comes up “Heads” and records $1 (loses $1) if the coin
comes up “Tails”. On the chart, the player records the outcome of each
flip by recording the flip number, the outcome as “H” or “T” and keeps
track of the cumulative fortune of the gambler so far. It is best to keep
these records in a neat chart, since we will refer to them later. Each
“gambler” should record 100 flips, which takes about 10 to 20 minutes.

For the homework, to turn in:

(a) Record the total number of heads, the total number of tails, and
   the difference of the number of heads and tails.

(b) Record whether the coin flip game reached “victory” +10 before
   reaching −10, or “ruin” or conversely reached ruin before reaching
   “victory”, or reached neither in 100 flips.

(c) Record the number of flips to first reach either +10 or −10 or
   state that the game reached neither.
(d) Record the total number of flips out of 100, i.e., the total time, that the number of Heads exceeded the number of Tails.

2. (a) For $T_0 = 10$ and $a = 20$, draw a graph of the probability of ruin as a function of the probability $q$.
   (b) For $a = 20$ and $q = 0.55$ draw a graph of the probability ruin as a function of $T_0$.
   (c) For $a = 20$ and $q = 0.45$ draw a graph of the probability of ruin as a function of $T_0$.

3. A gambler starts with $2 and wants to win $2 more to get to a total of $4 before being ruined by losing all his money. He plays a coin-flipping game, with a coin that changes with his fortune.
   (a) If the gambler has $2 he plays with a coin that gives probability $p = 1/2$ of winning a dollar and probability $q = 1/2$ of losing a dollar.
   (b) If the gambler has $3 he plays with a coin that gives probability $p = 1/4$ of winning a dollar and probability $q = 3/4$ of losing a dollar.
   (c) If the gambler has $1 he plays with a coin that gives probability $p = 3/4$ of winning a dollar and probability $q = 1/4$ of losing a dollar.

   Use “first step analysis” to write three equations in three unknowns (with two additional boundary conditions) that give the probability that the gambler will be ruined. Solve the equations to find the ruin probability.

4. A gambler plays a coin-flipping game in which the probability of winning on a flip is $p = 0.4$ and the probability of losing on a flip is $q = 1 - p = 0.6$. The gambler wants to reach the victory level of $16 before being ruined with a fortune of $0. The gambler starts with $8, bets $2 on each flip when the fortune is $6,$8,$10 and bets $4 when the fortune is $4 or $12. Compute the probability of ruin in this game.

5. Use the ruin probability notation to show that in a random walk starting at the origin the probability of reaching the point $a > 0$ before the random walk returns to the origin is $p(1 - q_1)$.  

2