1. Consider a stock whose price today is $50. Suppose that over the next year, the stock price can either go up by 6%, or down by 3%, so the stock price at the end of the year is either $53 or $48.50. The continuously compounded interest rate on a $1 bond is 4%. If there also exists a call option on the stock with an exercise price of $50, then what is the price of the call option? Also, what is the replicating portfolio?

2. A stock price is currently $50. it is known that at the end of 6 months, it will either be $60 or $42. The risk-free rate of interest with continuous compounding on a $1 bond is 10% per year. Calculate the value of a 6-month European call option on the stock with strike price $48 and find the replicating portfolio.

3. Consider a three-time-stage example. The first time interval is a month, then the second time interval is two months, finally, the third time interval is a month again. A stock starts at 50. In the first interval, the stock can go up by 10% or down by 3%, in the second interval the stock can go up by 5% or down by 5%, finally in the third time interval, the stock can go up by 6% or down by 3%. The continuously compounded interest rate on a $1 bond is 2% in the first period, 3% in the second period, and 4% in the third period. Find the price of a call option with exercise price 50, with exercise date at the end of the 4 months. Also, find the replicating portfolio at each node.
4. A long strangle option pays \(
\max(K_1 - S, 0, S - K_2)\) if it expires when the underlying stock value is \(S\). The parameters \(K_1\) and \(K_2\) are the lower strike price and the upper strike price, and \(K_1 < K_2\). A stock currently has price $100 and goes up or down by 20\% in each time period. What is the value of such a long strangle option with lower strike 90 and upper strike 110 at expiration 2 time units in the future? Assume a simple interest rate of 10\% in each time period.

5. Your friend, the financial analyst comes to you, the mathematical economist, with a proposal: “The single period binomial pricing is all right as far as it goes, but it certainly is simplistic. Why not modify it slightly to make it a little more realistic? Specifically, assume the stock can take three values at time \(T\), say it goes up by a factor \(U\) with probability \(p_U\), it goes down by a factor \(D\) with probability \(p_D\), where \(D < 1 < U\) and the stock stays somewhere in between, changing by a factor \(M\) with probability \(p_M\) where \(D < M < U\) and \(p_D + p_M + p_U = 1\).” The market contains only this stock, a bond with a continuously compounded risk-free rate \(r\) and an option on the stock with payoff function \(f(S_T)\). Make a mathematical model based on your friend’s suggestion and provide a critique of the model based on the classical applied mathematics criteria of existence of solutions to the model and uniqueness of solutions to the model.