1. Consider the hypothetical country of Elbonia, where the government has declared a “currency band” policy, in which the exchange rate between the domestic currency, the Elbonian Bongo Buck, denoted by EBB, and the US Dollar is guaranteed to fluctuate in a prescribed band, namely:

\[ 0.95 \text{USD} \leq \text{EBB} \leq 1.05 \text{USD} \]

for at least one year. Suppose also that the government has issued 1-year notes denominated in the EBB that pay a continuously compounded interest rate of 20%. Assuming that the corresponding continuously compounded interest rate for US deposits is \( \% \), show that there is an arbitrage opportunity.

**Solution:**

We start by shorting (borrowing) \( X \) USD and buying the EBB. We assume that this occurs in the worst case, which is when the EBB is most expensive in USD. Then we can purchase

\[ \frac{X \text{ USD}}{1.05 \text{ USD/EBB}}. \]

Now we invest these EBB in the 20% bonds, so at the end of a year, our total of EBB have grown to

\[ \frac{X}{1.05} \times \exp(0.20 \cdot 1). \]
Now we exchange these EBB back to USD. Again, we assume that this happens at the worst possible rate, when the EBB is least dear in USD, so that we receive
\[
\frac{\exp(0.30)X}{1.05} \text{USD} \frac{0.95\text{USD}}{\text{EBB}}.
\]
Then we pay the loan, which was at the 4\% rate for a total of \(\exp(0.04 \cdot 1)X\), so the amount left is
\[
\frac{\exp(0.20)X}{1.05} \times 0.95 - \exp(0.04)X = 0.064268X.
\]
I have carried the calculations at 5 significant digits, but the precision of the currency exchange rates is only given to 2 significant digits, so I will say that we make a profit of a little more than $0.064 per dollar borrowed, exchanged, and invested.

2. Consider a market that has a security and a bond so that money can be borrowed or loaned at an annual interest rate of \(r\) compounded continuously. At the end of a time period \(T\), the security will have increased in value by a factor \(U\) to \(SU\), or decreased in value by a factor \(D\) to value \(SD\). Show that a forward contract with strike price \(k\) that, is, a contract to buy the security which has potential payoffs \(SU - k\) and \(SD - k\) should have the strike price set at \(S \exp(rT)\) to avoid an arbitrage opportunity.

**Solution** We imagine that we start with no portfolio, no security and no cash.

Suppose that the strike price \(k\) is set so that \(k > S_0 \exp(rT)\). Then the strategy is to borrow \(S_0\), buy the security at current price \(S_0\) and enter into a forward contract to sell the security at strike price \(k\). Then at time \(T\), sell the security at price \(k\), deliver it, and pay back the bond loan in the amount \(S_0 \exp(rT)\), ending in the same state that we started in, no security, no cash and making a risk free profit of \(k - S_0 \exp(rT)\) on the deal.

Suppose instead that the strike price \(k\) is set so that \(k < S_0 \exp(rT)\). Then the strategy is to loan (short) \(S_0\) in bonds, and enter into that forward contract to buy the security at strike price \(k\). Then at time
$T$, cash in the bonds, get $S_0 \exp(rT)$, and use it to buy the security at price $k$, thereby making a risk free profit of $S_0 \exp(rT) - k$ on the deal. Note that the factors $U$ and $D$ do not enter into this calculation, they are irrelevant information.