Math 489/889
Stochastic Processes and
Advanced Mathematical Finance
Homework 10

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December 6, 2010

1. Simulate the solution of the stochastic differential equation
   \[ dX(t) = X(t)dt + 2X(t)dW \]
   on the interval \([0, 1]\) with initial condition \(X(0) = 1\) and step size \(\Delta t = 1/10\). Use the results of your coin-tossing record which was rescaled to \(\hat{W}_{10}(10t)/\sqrt{10}\) as an approximation of \(W(t)\) to create the 10 increments \(dW\) needed for the simulation.

2. Find the solution of the stochastic differential equation
   \[ dY(t) = Y(t)dt + 2Y(t)dW \]
   by guessing and checking with Itô’s Formula.

3. Find the mode (the value of the independent variable with the highest probability) of the lognormal probability density function. (Use parameters \(\mu\) and \(\sigma\).)

4. (a) Suppose you buy a stock (or more properly an index fund) whose value \(S(t)\) is described by the stochastic differential equation \(dS = 0 \cdot S dt + \sigma \cdot S dW\) corresponding to a zero compounded growth and pure market fluctuations proportional to the stock value. What are your chances to double your money?
(b) Suppose you buy a stock (or more properly an index fund) whose value \( S(t) \) is described by the stochastic differential equation \( dS = \frac{\sigma^2}{2} \cdot S \, dt + \sigma \cdot S \, dW \) corresponding to a non-zero compounded growth and pure market fluctuations proportional to the stock value and the two parameter values are coincidentally connected in their value. What are your chances to double your money?