Chain Rule Short Cuts

In class we applied the chain rule, step-by-step, to several functions. Here is a short list of examples.

1. Powers of functions

The rule here is

$$\frac{d}{dx}u(x)^a = au(x)^{a-1}u'(x) \tag{1}$$

So if

$$f(x) = (x + \sin x)^5,$$

then

$$f'(x) = 5(x + \sin x)^4\,(1 + \cos x).$$

The rule (1) is useful when differentiating reciprocals of functions. If $a = -1$ we get

$$\frac{d}{dx}\frac{1}{u(x)} = -\frac{u'(x)}{u(x)^2}.$$

You could also have derived this using the quotient rule.

2. Exponentials

For $a > 0$,

$$\frac{d}{dx}a^{u(x)} = a^{u(x)}\,u'(x)\,\ln a, \tag{2}$$

So if

$$g(x) = 3^{x^2-4x},$$

then

$$g'(x) = 3^{x^2-4x}\,(2x - 4)\,\ln 3.$$

3. The Natural logarithm of a function

The chain rule in this case says that

$$\frac{d}{dx}\ln u(x) = \frac{1}{u(x)}u'(x) \tag{3}$$

So if

$$f(x) = \ln(\sin x),$$

then

$$f'(x) = \frac{1}{\sin x}\,\cos x.$$

4. Trigonometric functions

We'll illustrate the chain rule with the cosine function.

$$\frac{d}{dx} \cos u(x) = -\sin u(x)\, u'(x) \tag{4}$$

Thus, if

$$\psi(x) = \cos\,(1 + x^3),$$

then

$$\psi'(x) = -3x^2\, \sin\,(1 + x^3).$$

Functions of the form $\sin u(x)$ and $\tan u(x)$ are handled similarly.

5. Inverse trigonometric functions

We'll use the arctan function. The chain rule tells us that

$$\frac{d}{dx} \arctan u(x) = \frac{1}{1 + u(x)^2}\, u'(x). \tag{5}$$

So if

$$\varphi(x) = \arctan\,(x + \ln x),$$

then

$$\varphi'(x) = \frac{1}{1 + (x + \ln x)^2} \left(1 + \frac{1}{x}\right).$$

Functions of the form $\arcsin u(x)$ and $\arccos u(x)$ are handled similarly.

Bear in mind that you might need to apply the chain rule as well as the product and quotient rules to to take a derivative. You might also need to apply the chain rule more than once. For example,

$$\frac{d}{dx} \sin\,(\ln\,(x - 2x^2)) = \cos\,(\ln\,(x - 2x^2))\, \frac{1}{x - 2x^2}\, (1 - 4x).$$