Duality Theory 2: The Dual Simplex Method for Maximization

1. **Notation:** Let τ be the vector of r_0 coefficients for the decision and slack variables. As usual, τ_0 and τ^* will be used in the initial and final tableaus respectively.

2. The simplex method starts with a suboptimal solution and moves toward optimality. The dual simplex method starts with an infeasible solution and moves toward feasibility. Here is an outline of the dual simplex method for a maximization problem.

a. Convert "$\geq$" functional constraints to the "$\leq$" kind by multiplying through by -1. Introduce slack variables.

b. Test for feasibility. Are the basic variables nonnegative? If they are, stop. If not, begin a new iteration.

c1. Determine the leaving variable. This is the negative basic variable of largest absolute value. The leaving variable's row is called the pivot row.

c2. Determine the entering variable. For each nonbasic variable with a negative coefficient in the pivot row, form the ratio

$$\frac{r_0 \text{ entry}}{\text{coefficent in pivot row}}. \tag{1}$$

The nonbasic variable for which the absolute value of this ratio is smallest is the entering variable. (By choosing the entering variable this way, you guarantee that the iteration won't leave you with new negative entries in τ.)

c3. Put the tableau in proper form.

c4. Test for feasibililty.

3. Suppose that you apply the dual simplex method to a problem of the form

$$\begin{cases} \text{Minimize} & y_0 = yb \\ \text{Subject To} & yA \geq c \\ & y \geq 0. \end{cases}$$

You'd first change it to

$$\begin{cases} \text{Maximize} & -y_0 = -yb \\ \text{Subject To} & -yA \leq -c \\ & y \geq 0. \end{cases}$$

Thus τ_0 in the initial tableau would contain no negative entries. Since the rule for selecting entering variables disallows the appearance of new negative entries in τ, the final tableau of the dual simplex method will identify a solution that is not only feasible, but optimal. We thus have three ways of solving the dual (D) of a standard primal (P).

a. You can put (D) in standard form with surplus and artificial variables, change the objective to "Maximize" etc., and then solve using the Big M method.

b. You can solve (P) and thus obtain an optimal solution y^* to (D) from the top row t^*.

c. You can apply the dual simplex method. By the above observation, the feasible tableau is optimal.