Math 221 Exam 1

Show your work. Justify your conclusions.

1. What is the order of the equation? Is it linear or nonlinear? If linear, is it homogeneous or inhomogeneous? If first order, is it separable or nonseparable? Autonomous or nonautonomous?
   [2] a. \((\cos^2 x)y'' + (\tan x)y' + x^4 y = e^{-x}\)
   [2] b. \(y'' + xy' = 0\)
   [2] c. \(y' = \sin (1 + xy^2 + x^3)\)
   [2] d. \(y' = \ln (1 + y^4)\)

[8] 2. Solve the initial value problem

\[
\begin{cases}
2yy' = x(x^2 - 16)^{-\frac{1}{2}}, \\
y(5) = 2.
\end{cases}
\]

[8] 3. Find the general solution to the equation \(2xy' + y = 10\sqrt{x}\).

[8] 4. A 400 gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at rate 5 gal/s, and the well-mixed brine in the tank flows out at rate 3 gal/s. How much salt will the tank contain when it is full of brine?

[6] 5. Find the general solution to the equation \(1 + ye^{xy} + (2y + xe^{xy})y' = 0\).

[6] 6. Find the general solution to the equation \(xy' + 6y = 3x y^\frac{3}{2}\).

[6] 7. A water tank has the shape obtained by revolving the parabola \(x^2 = by\) around the \(y\) axis. The water depth is 4 ft at 12 noon when a circular plug is the bottom of the tank is removed. At 1 P.M. the depth of the water is 1 ft. Find the depth \(y(t)\) of the water remaining after \(t\) hours.

8. Consider the Cauchy problem

\[
(P) \begin{cases}
y' = f(x, y), \\
y(x_0) = y_0.
\end{cases}
\]


[3] b. Find two solutions to \((P)\) with \(x_0 = 1, y_0 = 0\) and \(f(x, y) = 3xy^\frac{3}{4}\). Does this contradict the theorem from part (a)? Justify your answer.

9. Consider the autonomous equation \(x' = x^2 - 5x + 4\).
   [2] b. Use the phase line to sketch the general solution.
   [2] c. Which (if any) of the equilibrium solutions are stable? Which are unstable?