

Outline for the 221H Final Exam

Ground Rules: As before, you'll have a choice of problems. You will be given a table of Laplace transforms. You can use any calculator you wish, but you'll have to justify your results.

Note: Nonlinear systems and Euler's method will *not* appear on the final exam.

First Order Equations

1. First-order, separable, equations.
2. First-order, linear equations.
3. Homogeneous equations, i.e. equations of the form $u' = f\left(\frac{u}{t}\right)$.
4. Exact equations.
5. Bernoulli equations.
6. The initial value problem for equations of the foregoing types.
7. Direction fields.
8. Phase line analysis of autonomous equations.
9. The existence-uniqueness theorem for the initial value problem.
10. The method of Picard iteration.

Second-Order Equations

1. The statements, meanings and proofs of the theorems (propositions 13, 15 and 16 from the second set of webnotes on linear equations) about Wronskians, fundamental sets and general solutions to second-order, linear, homogeneous equations.
2. Finding a fundamental set for a homogeneous equation with constant coefficients.
3. Finding a fundamental set of series solutions.
4. Finding a fundamental set for a Cauchy-Euler equation.
5. Derivation and use of the reduction of order formula.
6. Derivation and use of the variation of parameters formula to solve the inhomogeneous equation.

Mathematical Modeling

1. Newton's law of cooling.
2. Derivation and use of the mass balance law for the chemostat.
3. Use and derivation, from Newton's law and conservation of energy, of equations of motion (e.g. the pendulum equation, body falling through a viscous fluid).
4. The logistic model of population growth.

Laplace Transforms

1. Calculation of Laplace transforms.
2. Properties of the Laplace transform: Shift, switching etc.
3. Inverting Laplace transforms: Partial fractions, the convolution theorem, etc.
4. Solving initial value problems with the Laplace transform.
5. Discontinuous and point sources: The Heaviside and Dirac delta functions. The physical meaning of the delta function. Solving initial value problems with Heaviside and delta functions.

Linear Systems of Ordinary Differential Equations

1. Matrices and determinants. Linear independence. Eigenvalues, eigenvectors. The trace and determinant.
2. Homogeneous and inhomogeneous n -dimensional linear systems. The existence-unicity theorem for the initial value problem.
3. The homogeneous system. Superposition, linearly independent solutions, the general solution. Results from lecture: Independence of solutions at one point implies their independence everywhere. Every solution can be written as a linear combination of n independent solutions.
4. The homogeneous, constant-coefficient, 2-dimensional system. Finding the general solution, solving the initial value problem. Distinct eigenvalues, the real and complex cases. The case of a single eigenvalue.
5. The phase plane. Phase portraits. Equilibria, linear orbits, nodes, saddle points, spirals and centers. Stable, asymptotically stable and unstable orbits. Classification of equilibria using the trace and determinant.
6. The inhomogeneous linear system. Fundamental matrices. The general solution. The variation-of-parameters solution formula. The initial value problem.