Math 208, Fall 2010, Exam 1

Do problems 1-4 and any two of 5-7.
Show your work, provide explanation where appropriate.
You don't need to simplify. You shouldn't need a calculator.

1. Let $\vec{u} = \langle 1, 2, -1 \rangle$, $\vec{v} = \langle 3, 1, 0 \rangle$ and $\vec{w} = \langle -2, 2, 1 \rangle$.

[2] **a**. Compute $\vec{v} \times \vec{w}$.

[2] **b**. Find a vector $\vec{n} \neq \vec{0}$ that is orthogonal to $\vec{v}$ and $\vec{w}$.

[2] **c**. Find the equation of the plane through $(1, -1, 2)$ with normal vector $\vec{n}$.

[2] **d**. Compute the area of the parallelogram formed by $\vec{v}$ and $\vec{w}$.

[2] **e**. Compute the volume of the parallelpiped formed by $\vec{u}$, $\vec{v}$ and $\vec{w}$.

2. Let $u(x, y) = \sin(x^2 - y)$.

[7] **a**. Compute the first partial derivatives of u.

[3] **b**. Compute the second-order partial derivatives of u.

3. Let $f(x, y, z) = xy + e^{yz}$, $P = (1, 2, 0)$ and $Q = (1.1, 1.9, .05)$

[4] **a**. Find the derivative of f at P in the direction $\vec{v} = \langle -2, 2, 1 \rangle$.

[4] **b**. In which direction is f increasing most rapidly at P? What is the derivative of f at P in that direction?

[3] **c**. Use the differential to estimate $\Delta f = f(Q) - f(P)$.

[3] **d**. Find a vector normal to the level surface $f(x, y, z) = 3$ at the point P.

[8] **4**. The voltage V, current I and resistance R in a circuit are related by Ohm's law, $V = IR$. Find dV/dt when $R = 600$ ohm, $I = .04$ amp, $dR/dt = .5$ ohm/sec, and $dI/dt = -.01$ amp/sec.

[4] **5**. Let $f(x, y)$ be a smooth function with $f(a, b) = c$. Show that $\nabla f(a, b)$ is normal to the level curve $f(x, y) = c$ at the point (a, b). (**Hint**: Let $(x(t), (y(t))$ be a parametrization of the level curve. Apply the chain rule to $f(x(t), y(t))$.)

[4] **6**. Find the local maxima, local minima and saddle points of $f(x, y) = 2xy - x^2 - 2y^2 + 3x + 4$.

[4] **7**. Find the points on the ellipsoid $x^2 + y^2 + 2z^2 = 24$ where $f(x, y, z) = x + y + 4z$ has its maximum and minimum values. (You don't have to sort the maximizers from the minimizers.)