Math 208
Polar coordinate iterated integrals

1. Set up and evaluate a double integral in polar coordinates for $\iint\limits_R x^2 y \, dA$, where R is the region inside $x^2 + y^2 = 4$ and above the line $y = -x$.

2. Set up and evaluate a double integral in polar coordinates for $\iint\limits_R (x + y) \, dA$, where R is the portion of the region inside the circle $x^2 + y^2 = 2y - 2x$ that is in the first quadrant. (Note the circle goes through the origin and has its center in quadrant II.)

3. Convert to polar coordinates: $\int_{-1}^{0} \int_{\sqrt{3}}^{\sqrt{4-x^2}} y(x^2 + y^2) \, dy \, dx$. Do not evaluate.

4. Use a double integral in polar coordinates to find $\int_R \frac{1}{\sqrt{x^2+y^2}} \, dA$, where R is one loop of the "four leaf clover" $r = 6\cos(2\theta)$.

5. Use a double integral in polar coordinates to find $\int_R y \, dA$, where R is the region below the x-axis, inside the circle $x^2 + y^2 = 4x$, and outside the circle $x^2 + y^2 = 4$.

6. Find the area inside the cardioid $r = 2 + 2\sin\theta$ and outside of the circle $r = 3$.

Answers:

1. $\frac{16\sqrt{2}}{15}$

2. $\frac{2}{3}$

3. $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \int_{\frac{\sqrt{3}}{\sin\theta}}^{2} r^4 \sin\theta \, dr \, d\theta$

4. 6

5. $-\frac{11}{3}$

6. $\frac{9}{2}\sqrt{3} - \pi$