Final Exam Outline

1. Vectors.

a. The magnitude and direction of a vector. Parallel vectors. Unit vectors.

b. Vector algebra. The zero vector.

c. The dot product. The angle between two vectors. Orthogonal vectors.

d. The cross product of two vectors. The magnitude and direction of the cross product. The area of a parallelogram. The volume of a parallelpiped.

2. Planes in $\mathbf{R}^3$.

a. The equation of the plane through a given point normal to a given vector.

b. The equation of the plane through three (non-colinear) points.

c. The equation of the plane, parallel to another, through a given point.

3. Functions of several variables.*

a. Functions taking $\mathbf{R}^2$ or $\mathbf{R}^3$ to $\mathbf{R}$. Their domains and ranges.

b. Graphs, contours, density plots and traces of functions of two variables.

c. Level surfaces of functions of three variables.

4. Partial derivatives.

a. Calculation of partial derivatives.

b. Higher-order partial derivatives.

c. Equality of certain mixed partial derivatives.

5. Linear approximation.

a. The tangent plane to the graph of a function of two variables.

b. Differentials. The principle of (local) linear approximation, versions I and II.

6. The chain rule.

a. The chain rule for various compositions: $z(t) = f(x(t), y(t))$, $u(x,t) = f(\theta(x,t))$, $w(p,q) = f(x(p,q), y(p,q))$, etc. Tree diagrams. Computation of second derivatives using the chain rule.

b. Implicitly defined functions. Partial derivatives of implicit functions.

7. Gradients and directional derivatives.

a. The gradient of a function of several variables.

b. The derivative of a function at a point p in a unit direction $\vec{u}$.

c. Properties of the gradient. Direction and rate of most rapid increase. The orthogonality of ∇g to the contour surfaces (or curves) of g.

8. Extrema of functions of several variables.

a. Relative (or local) extrema and saddle points.

b. Critical points.

c. The second derivative test for functions of two variables.

9. Constrained optimization.

a. The Lagrange multiplier method for identifying constrained extrema.

10. Double integrals.

- **a**. The double integral over a rectangular and nonrectangular regions.
- **b**. Iterated integrals over rectangles and regions bounded by curves. Fubini's theorem. Changing the order of integration.
- **c**. Applications of the double integral: Area, volume, moments, mass.

11. Triple integrals.

- **a**. Triple integrals over boxes and more general regions.
- **b**. Iterated integrals over boxes. Iterated integrals over regions bounded by surfaces. with $dV = dx\,dA$, $dV = dy\,dA$ or $dV = dz\,dA$. Fubini's theorem. Changing the order of integration.
- **c**. Applications of the triple integral: Volume, moments, mass.

12. Change of variable in double and triple integrals.

- **b**. Integrals in polar, cylindrical and spherical coordinates.

13. Vector fields.

- **a**. Conservative, or gradient fields. Potential functions. Inverse square vector fields.
- **b**. The curl of a vector field. The curl test for conservative fields.
- **c**. Finding the the potential of a conservative field.

14. Line integrals.

- **a**. Line integrals of functions.
- **b**. Line integrals of vector fields over oriented curves. The line integral as work done in traversing a path in a force field. Circulation.
- **c**. Evaluation of line integrals.

15. Path independent line integrals.

- **a**. Path independent line integrals and conservative vector fields.
- **b**. The fundamental theorem of line integration.
- **c**. Line integrals over closed paths.

16. Surface integrals.

- **a**. The areas of surfaces given paramterically or explicitly.
- **b**. Integrals of functions over surfaces.
- **c**. Integrals of vector fields over surfaces (flux integrals).
- **d**. Calculation of flux integrals.

17. The divergence theorem.

- **a**. The divergence of a vector field.
- **b**. Applications of the divergence theorem: Computing flux integrals over closed surfaces.

18. Stokes' theorem and Green's theorem.

- **a**. Stokes' theorem. Using Stokes' theorem to compute line and surface integrals.
- **b**. Green's theorem as the two-dimensional version of Stokes' theorem. Using Green's theorem to compute line and double integrals on the xy-plane.