

**Math 208**  
**Gradient applications additional problems**

1. Let  $f(x, y) = -0.25x^2 - 0.25y^2$ .
  - a) Find  $\nabla f(-3, 4)$ .
  - b) Find an equation for the level curve of  $f$  going through the point  $(-3, 4)$ .
  - c) Find an equation for the tangent line at  $(-3, 4)$  to the equation found in (b).
  - d) Find a system of parametric equations and a non-parametric equation for the normal line to the level curve of  $f$  going through the point  $(-3, 4)$ .
  - e) In which direction does  $f$  decrease most rapidly at  $(-3, 4)$ , and what is the rate of change of  $f$  at  $(-3, 4)$  in that direction?
  - f) What is the directional derivative of  $f$  at  $(-3, 4)$  in the direction from there toward  $(0, 1)$ ?
  - g) Find all unit vectors  $\mathbf{u}$  such that  $f_{\mathbf{u}}(-3, 4) = 0$ .
  - h) Find the unit vectors in all directions for which the rate of change of  $f$  at  $(-3, 4)$  equals 2.
  
2. Let  $f(x, y, z) = x^3z - 2yz^2 - 2z$ . Simplify all numbers in this problem.
  - a) Find  $\nabla f(2, -1, 3)$ .
  - b) Find the rate of change of  $f$  at  $(2, -1, 3)$  in the direction from there toward  $(-4, 1, 6)$ .
  - c) Find an equation for the tangent plane to the level surface  $f(x, y, z) = 36$  at the point  $(2, -1, 3)$ .
  - d) Find parametric equations for the normal line to the level surface  $f(x, y, z) = 36$  at the point  $(2, -1, 3)$ .
  - e) Find a unit vector in the direction in which  $f$  increases most rapidly at  $(2, -1, 3)$ , and the rate of change of  $f$  in that direction.

Gradient applications handout answers:

1. a)  $\langle 1.5, -2 \rangle$       b)  $-0.25x^2 - 0.25y^2 = -6.25$ , or  $x^2 + y^2 = 25$   
c)  $1.5(x+3) - 2(y-4) = 0$  or equivalent, such as  $4y - 3x = 25$   
d) Parametric:  $x = -3 + 1.5t$  and  $y = 4 - 2t$   
Non-parametric:  $\frac{x+3}{1.5} = \frac{y-4}{-2}$  or equivalent such as  $4x + 3y = 0$   
e) Direction is direction of  $\langle -1.5, 2 \rangle$ , rate of change is  $-2.5$   
f)  $\frac{3.5}{\sqrt{2}}$       g)  $\mathbf{u} = \langle 0.8, 0.6 \rangle$  or  $\mathbf{u} = \langle -0.8, -0.6 \rangle$   
h)  $\langle 0, -1 \rangle$  and  $\langle 0.96, -0.28 \rangle$  are the only such unit vectors
  
2. a)  $\langle 36, -18, 18 \rangle = 18\langle 2, -1, 1 \rangle$       b)  $-28\frac{2}{7}$   
c)  $2x - y + z = 8$  (or equivalent)  
d)  $x = 2 + 2t$ ,  $y = -1 - t$ ,  $z = 3 + t$  (or equivalent)  
e) unit vector  $\frac{\langle 2, -1, 1 \rangle}{\sqrt{6}}$ , rate of change  $18\sqrt{6}$