IMMERSE 2010
Algebra Course
Problem Set 6

Work all of the following problems, and turn in a set of solutions as a ring.

1. Classify each element of $\mathbb{Z}_{18}$ as a unit or a zero divisor. For those elements that are units, exhibit a multiplicative inverse.

2. Find an integer $n$ such that the ring $\mathbb{Z}_n$ does not have the following properties of the ring of integers:
   
   (a) $a^2 = a$ implies $a = 0$ or $a = 1$.
   (b) $ab = 0$ implies $a = 0$ or $b = 0$.
   (c) $ab = ac$ and $a \neq 0$ implies $b = c$.

3. Show that the three properties in the previous question are valid for $\mathbb{Z}_p$, where $p$ is prime.