Work all of the following problems, and turn in a set of solutions as a ring. Each group member should be responsible for TeX-ing at least one problem.

1. Prove that $\mathbb{Z}[x]$ is not a PID.

2. Show that if $f(x) \in \mathbb{Z}[x]$ is a nonconstant polynomial that is irreducible over $\mathbb{Z}$, then $f(x)$ is primitive.

3. Determine whether each of the following polynomials is irreducible over $\mathbb{Q}[x]$:
   
   (a) $x^5 + 9x^4 + 12x^2 + 6$
   (b) $x^3 - 11x^2 + 45$
   (c) $x^4 - x^2 + 3$

4. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \in \mathbb{Z}[x]$, where $a_n \neq 0$. Prove that if $r$ and $s$ are relatively prime integers and $f(r/s) = 0$, then $r|a_0$ and $s|a_n$.

5. Let $F$ be a field, and $p(x) \in F[x]$ be irreducible over $F$. If $E$ is some field containing $F$, and there exists $a \in E$ such that $p(a) = 0$, show that the map

   $\phi : F[x] \to E$

   $f(x) \mapsto f(a)$

   is a ring homomorphism with kernel $(p(x))$. [Note: $(p(x)) = \{f(x)p(x) | f(x) \in F[x]\}$ is the ideal generated by $p(x)$.]