221 Differential Equations
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Practice Problems for Test 2

1. Verify that the given functions are solutions of the DE.

$$y''' + 2y'' - y' - 2y = 0, \quad e^t, e^{-t}, e^{-2t}.$$

Determine the Wronskian of each pair of two functions.

2. Find the general solution of the given DE

$$2y''' - 4y'' - 2y' + 4y = 0.$$

3. Let the linear differential operator L be defined by

$$L[y] = Ay^{(4)} - 5y^{(3)} + By'' + y,$$

where A, B are real constants.

(a) Find $L[t^4]$.
(b) Find $L[e^{rt}]$.
(c) Write the characteristic equation for $L[y] = 0$.

4. Use the method of reduction of order to solve the DE:

$$(t-1)y'' - ty' + y = 0, \quad t > 1,$$

knowing that a particular solution is $y_1(t) = e^t$. (*Hint:* Use the substitution $y = y_1(t)v(t)$ and derive a DE for v.

5. Find the solution of the initial value problem

$$u'' + u = F(t), \quad u(0) = 0, \quad u'(0) = 0,$$

where

$$F(t) = \begin{cases} F_0 t, & 0 \le t \le \pi, \\ F_0(2\pi - t), & \pi < t \le 2\pi, \\ 0, & 2\pi < t. \end{cases}$$

Hint: Treat each time interval separately, and match the solutions in the different intervals by requiring that u and u' be continuous functions of t.

6. Use the method of variation of parameters to determine the solution of the given IVP:

$$y'' + y = \sec t; \quad y(0) = 2, \quad y'(0) = 1.$$

7. Use the method of undetermined coefficients to solve the following DEs:

(a) $2y'' + 3y' + y = t^2 + 3\sin t$

(b) $y'' + 2y' + 5y = 4e^{-t}\cos 2t$

8. If an undamped spring-mass system with a mass that weighs 6lb and a spring constant 1lb/in is suddenly set in motion at $t = 0$ by an external force of $4 \cos t$ lb, determine the position of the mass at any time.

9. In the absence of damping the motion of a spring-mass system satisfies the initial value problem

$$mu'' + ku = 0, \quad u(0) = a, \quad u'(0) = b.$$

   (a) Show that the kinetic energy initially imparted to the mass is $mb^2/2$ and that the potential energy initially stored in the spring is $ka^2/2$, so that initially the total energy in the system is $(ka^2 + mb^2)/2$.

   (b) Solve the given initial value problem.

   (c) Using the solution in part b), determine the total energy in the system at any time. Your result should confirm the principle of conservation of energy for this system.