Math 970 Homework 9

Due Thursday, Nov. 20

38. Show that if X is limit point compact, and A is a closed subset of X, then A is limit point compact.

39. Give an example of a limit point compact space X and a continuous function $f : X \to Y$ for which $f(X) \subseteq Y$ is not limit point compact.

Note: for the purposes of the following problems, " regular" means points and (disjoint) closed sets can be separated with open sets, "normal" means disjoint closed sets can be separated, T_3 means T_1 and regular, and T_4 means T_1 and normal.

40. Show by examples that the continuous image of T_3 need not be T_3, and that the continuous image of a non-T_3 space can be T_3!

41. Show that every closed subset of a normal space is normal, and that every closed subset of a T_4 space is T_4.

42. Show that for any collection $X_\alpha \neq \emptyset$ of topological spaces, if $\prod_\alpha X_\alpha$ is T_4 in the product topology, then X_α is T_4 for all α.
    (Hint: embed X_α in $\prod_\alpha X_\alpha$ as a closed subset!)