Math 970 Homework # 8

Due: November 11

33. Show that if $(X, \mathcal{T})$ is compact, and $\mathcal{T}' \subseteq \mathcal{T}$, then $(X, \mathcal{T}')$ is compact.

34. Show that if $(X, \mathcal{T})$ is a topological space and A, B are compact subsets of X, then $A \cup B$ is compact.

35. Give an example of a space $(X, \mathcal{T})$ and subsets $A, B \subseteq X$ so that A and B are compact but $A \cap B$ is not.

    (Note: your space X cannot be Hausdorff....)

36. Let $X = \mathbb{R}$ with the infinite ray topology

    $$\mathcal{T} = \{(a, \infty) : a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$$

    Show that $A = \{0\}$ is a compact subset of X, but its closure $\bar{A}$ isn't.

37. Show that if $(X, \mathcal{T})$ is a Hausdorff space and $A, B \subseteq X$ are disjoint compact subsets of X, then there are subsets $\mathcal{U}, \mathcal{V} \in \mathcal{T}$ so that $A \subseteq \mathcal{U}$, $B \subseteq \mathcal{V}$, and $\mathcal{U} \cap \mathcal{V} = \emptyset$.