30. Show that if $X_\alpha, \alpha \in I$ are all path-connected, then so is $\prod_{\alpha \in I} X_\alpha$, if we use the product topology.

31. Show that if $A_\alpha \subseteq X, \alpha \in I$ are all path-connected, and $\bigcap_{\alpha \in I} A_\alpha \neq \emptyset$, then $\bigcup_{\alpha \in I} A_\alpha$ is path-connected.

32. Show that if $C \subseteq \mathbb{R}^3$ is countable, then $\mathbb{R}^3 \setminus C$ is path-connected. (Hint: a plane in $\mathbb{R}^3$ will hit $C$ in how many points?)