Math 970 Homework # 5

Due: Oct. 9

22. Show that if X is a space with topology generated by a basis $\mathcal{B}$, then X is Hausdorff if and only if for every $x, y \in X$ with $x \neq y$, there are $B, B' \in \mathcal{B}$ with $x \in B$, $y \in B'$ and $B \cap B' = \emptyset$.

23. Show that if $\mathcal{T}$ is the usual topology on $\mathbb{R}$, the space $X = \mathbb{R} \cap \{*\}$, with topology generated by the basis $\mathbb{B} = \mathcal{T} \cup \{(U \setminus 0) \cup \{*\} : U \in \mathcal{T} \text{ and } 0 \in U\}$ is not Hausdorff, but every one-point subset of X is closed. [FYI: X is called the *line with two origins.*]

24. Show that the line with two origins is the quotient of two disjoint copies of $\mathbb{R}$ (think: $\mathbb{R} \times \{0, 1\}$). Conclude that the quotient of a Hausdorff space need not be Hausdorff.

25. Show that the quotient space obtained by the equivalence relation $\sim$ on $[0, 1] \times [0, 1]$ generated by (i.e., add $a \sim a$, and $a \sim b$ whenever $b \sim a$, and any relation that transitivity would *force* on you)

$$(0, y) \sim (1, y) \text{ for all } y \in [0, 1] \text{ and } (x, 0) \sim (x, 1) \text{ for all } x \in [0, 1]$$

admits a continuous bijection to $S^1 \times S^1$.

26. Find an example of subspaces $A, B \subseteq \mathbb{R}$ (giving $\mathbb{R}$ the usual topology) for which there is a continuous bijection

$$f : A \to B$$

whose inverse is **not** continuous.