Math 970 Homework # 4

Due: Oct. 2

17. For $A, B \subseteq X$ with $(X, \mathcal{T})$ a topological space, if A is open in X and B is closed in X, then $A \setminus B$ is open and $B \setminus A$ is closed.

18. Show that if $A, B \subseteq X$, then
    (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
    (b) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$, but that equality does not hold in general,
    (c) $\overline{A \setminus B} \supseteq \overline{A} \setminus \overline{B}$, but that equality does not hold in general.

19. Show that if $A \subseteq X$ and X has two topologies $\mathcal{T} \subseteq \mathcal{T}'$, then if $x \in X$ is a limit point of A w.r.t. $\mathcal{T}'$, then it is a limit point of A w.r.t. $\mathcal{T}$.

20. Show that if $A_i \subseteq X_i$ for all $i \in I$, then
$$\overline{\prod_i A_i} = \prod_i \overline{A_i} \subseteq \prod_i X_i$$
for <u>both</u> the product and box topologies.

21. Find the closure of the set $(0, 1) \subseteq \mathbb{R}$, when $\mathbb{R}$ has the
    (a) finite complement topology
    (b) infinite (open) ray to the right topology
    (c) discrete topology
    (d) *lower limit topology*, generated by the basis $\mathcal{B} = \{[a, b) : a, b \in \mathbb{R}\}$