Math 970 Homework # 4
Due: Oct. 2

17. For $A, B \subseteq X$ with $(X, T)$ a topological space, if $A$ is open in $X$ and $B$ is closed in $X$, then $A \setminus B$ is open and $B \setminus A$ is closed.

18. Show that if $A, B \subseteq X$, then
   
   (a) $\overline{A \cup B} = \overline{A \cup B}$
   
   (b) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$, but that equality does not hold in general,

   (c) $\overline{A \setminus B} \supseteq \overline{A} \setminus \overline{B}$, but that equality does not hold in general.

19. Show that if $A \subseteq X$ and $X$ has two topologies $T \subseteq T'$, then if $x \in X$ is a limit point of $A$ w.r.t. $T'$, then it is a limit point of $A$ w.r.t. $T$.

20. Show that if $A_i \subseteq X_i$ for all $i \in I$, then

   \[
   \prod \overline{A_i} = \prod \overline{A_i} \subseteq \prod X_i
   \]

   for both the product and box topologies.

21. Find the closure of the set $(0, 1) \subseteq \mathbb{R}$, when $\mathbb{R}$ has the

   (a) finite complement topology
   
   (b) infinite (open) ray to the right topology
   
   (c) discrete topology
   
   (d) lower limit topology, generated by the basis $\mathcal{B} = \{[a, b) : a, b \in \mathbb{R}\}$