11. Let \((X, T)\) be a topological space, \(B \subseteq X\) a subset, and \(T_B\) the subspace topology on \(B\). If \(A \subseteq B\), show that the subspace topology that it inherits from \(B\) is the same as the subspace topology that it inherits from \(X\).

12. Show that if \(A \subseteq X\) and \((X, T)\) is Hausdorff, then the subspace topology on \(A\) is Hausdorff.

13. Show that if \((X, d)\) and \((Y, d')\) are metric spaces, then the product topology on \(X \times Y\) is metrizable. [There are lots of (correct) choices of metric on \(X \times Y\); you can take your cue from \(\mathbb{R}^2\).]

14. Show that if \((X, T), (Y, T')\) are topological spaces and \(x_0 \in X\), then the function
\[
\iota_{x_0} : Y \to X \times Y, \quad \iota_{x_0}(y) = (x_0, y)
\]
is continuous.

15. Show that if \((X, d)\) is a metric space, then the metric \(d : X \times X \to \mathbb{R}\) is continuous (where \(X \times X\) has the product topology). Show, further, that the metric topology \(T\) is the coarsest topology on \(X\) for which \(d\) is continuous.

(Hint: show that if \(T' \subset T\), then \(N_d(x_0, \epsilon) \notin T'\) for some \(x_0\) and \(\epsilon\); now look at problem # 14.)

16. Show that, if \(X\) is an infinite set, then the finite complement topology \(T_f\) on \(X \times X\) is not a product topology, i.e., there do not exist topologies \(T, T'\) on \(X\) whose product topology is \(T_f\). On the other hand, if \(X\) is finite, show that \(T_f\) on \(X \times X\) is a product topology.

(Hint: the basis for the product topology would have to be \(\subseteq T_f\)...)

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