Math 970 Homework # 3

Due: Sept. 25

11. Let $(X, \mathcal{T})$ be a topological space, $B \subseteq X$ a subset, and $\mathcal{T}_B$ the subspace topology on B. If $A \subseteq B$, show that the subspace topology that it inherits from B is the <u>same</u> as the subspace topology that it inherits from X.

12. Show that if $A \subseteq X$ and $(X, \mathcal{T})$ is Hausdorff, then the subspace topology on A is Hausdorff.

13. Show that if (X, d) and (Y, d') are metric spaces, then the product topology on $X \times Y$ is metrizable. [There are lots of (correct) choices of metric on $X \times Y$; you can take your cue from $\mathbb{R}^2$.]

14. Show that if $(X, \mathcal{T}), (Y, \mathcal{T}')$ are topological spaces and $x_0 \in X$, then the function
$$\iota_{x_0} : Y \to X \times Y \ , \ \iota_{x_0}(y) = (x_0, y)$$
is continuous.

15. Show that if (X, d) is a metric space, then the metric $d : X \times X \to \mathbb{R}$ is continuous (where $X \times X$ has the product topology). Show, further, that the metric topology $\mathcal{T}$ is the <u>coarsest</u> topology on X for which d is continuous.

    (Hint: show that if $\mathcal{T}' \subsetneq \mathcal{T}$, then $N_d(x_0, \epsilon) \notin \mathcal{T}'$ for some x_0 and ϵ ; now look at problem # 14.)

16. Show that, <u>if</u> X is an infinite set, then the finite complement topology $\mathcal{T}_f$ on $X \times X$ is <u>not</u> a product topology, i.e., there do not exist topologies $\mathcal{T}, \mathcal{T}'$ on X whose product topology is $\mathcal{T}_f$. On the other hand, if X is finite, show that $\mathcal{T}_f$ on $X \times X$ <u>is</u> a product topology.

    (Hint: the <u>basis</u> for the product topology would have to be $\subseteq \mathcal{T}_f$ ...)