Math 970 Homework 11

Due Tuesday, Dec. 9

48. Show that if X is Hausdorff and $f{:}X \to X$ is continuous, then the fixed point set
$$\text{Fix}(f) = \{x \in X : f(x) = x\}$$
of f is a closed subset of X.

A subset $A \subseteq X$ is a <u>retract</u> of X if there is a continuous map $r{:}X \to A$ with $r \circ i = Id$, i.e., $r(a) = a$ for all $a \in A$. The map r is called a <u>retraction</u>.

49. Show that if X is Hausdorff and $A \subseteq X$ is a retract of X, then A is closed.
(Hint: show that A is the fixed point set of some map!)

50. Show that if $r : X \to A$ is a retraction and $a \in A$, then
$$r_*{:}\pi_1(X, a) \to \pi_1(A, a)$$
is a surjective homomorphism.

51. Show that if $a \in A \subseteq X$, $\pi_1(X, a) = \{1\}$, and $f{:}(A, a) \to (Y, b)$ is continuous, then if f extends to a continuous map $g{:}X \to Y$ (i.e., $g|_A = f$), then $f_*{:}\pi_1(A, a) \to \pi_1(Y, b)$ is the trivial homomorphism.
(The contrapositive of the last part of this statement sounds stronger....)

52. A space X is <u>contractible</u> if the identity map $I{:}X \to X$ is homotopic to the constant map $c(x) = x_0$. Show that if X is contractible then any two maps $f, g{:}Y \to X$ are homotopic. Show that this implies that $\pi_1(X, x_0) = \{1\}$.