Math 970 Homework 10

Due Tuesday, Dec. 2

43. Show that a compact metric space (X, d) is second countable.
(Hint: look at $\{N_d(x, 1/n) : x \in X\}$ for each n .)

44. Show that a closed subset of a Lindelöf space is Lindelöf.

45. Show by example that a closed subset of a separable space need not be separable.

46. Show that the continuous image of a separable space is separable, and the continuous image of a Lindelöf space is Lindelöf..

47. Show that if $(X, \mathcal{T}(\mathcal{C}))$ is second countable (with $\mathcal{C} = \{C_n\}_{n=1}^{\infty}$ countable), then <u>every</u> basis $\mathcal{B}$ for $\mathcal{T} = \mathcal{T}(\mathcal{C})$ contains a countable basis $\mathcal{B}' \subseteq \mathcal{B}$.
(Hint: look at all $B \in \mathcal{B}$ with $C_m \subseteq B \subseteq C_n$ for some m, n ; then pick (at most) one for each pair...)