Math 970 Homework 10
Due Tuesday, Dec. 2

43. Show that a compact metric space \((X, d)\) is second countable.
   (Hint: look at \(\{N_d(x, 1/n) : x \in X\}\) for each \(n\).)

44. Show that a closed subset of a Lindelöf space is Lindelöf.

45. Show by example that a closed subset of a separable space need not be separable.

46. Show that the continuous image of a separable space is separable, and the continuous image of a Lindelöf space is Lindelöf.

47. Show that if \((X, \mathcal{T}(\mathcal{C}))\) is second countable (with \(\mathcal{C} = \{C_n\}_{n=1}^{\infty}\) countable), then every basis \(\mathcal{B}\) for \(\mathcal{T} = \mathcal{T}(\mathcal{C})\) contains a countable basis \(\mathcal{B}' \subseteq \mathcal{B}\).
   (Hint: look at all \(B \in \mathcal{B}\) with \(C_m \subseteq B \subseteq C_n\) for some \(m, n\); then pick (at most) one for each pair...)