Math 872 Algebraic Topology

Problem Set # 9

Starred (*) problems due Tuesday, April 24

44. Compute the (cellular) homology groups of the quotient spaces $\mathbb{R}P^n/\mathbb{R}P^m$ for $m < n$. (Here we take $\mathbb{R}P^m$ to be the image of $S^n \cap (\mathbb{R}^{m+1} \times \{0\}) \subseteq \mathbb{R}^{n+1}$ under the antipodal map.)

45. Show that if $A \subseteq X$ both have $\oplus_i H_i(A), \oplus_i H_i(X)$ finitely generated, and A has an open neighborhood that deformation retracts to it, then $\oplus_i H_i(X/A)$ is finitely generated and $\chi(X) = \chi(A) + \chi(X/A) - 1$.

(*) 46. Show that $\widetilde{H}_i(S^n \setminus X) \cong \widetilde{H}_{n-i-1}(X)$ when X is homeomorphic to a finite commected graph. (Hint: prove it first for the case that X is a tree, then (inductively) add one edge at a time.)

47. For X a finite CW-complex and F a field, show that the Euler characteristic of X can be computed as $\chi(X) = \sum_i (-1)^i \dim_F(H_i(X; F))$.

(*) 48. Show that if X is a space with $H_k(X; \mathbb{Z}) \cong \mathbb{Z}_n$, then $H_{k+1}(X; \mathbb{Z}_n) \neq 0$.

(Hint: look at the LEHS induced by the SES of coefficient groups

$$0 \to \mathbb{Z} \xrightarrow{\times n} \mathbb{Z} \to \mathbb{Z}_n \to 0 \text{ .)}$$