Math 872 Algebraic Topology

Problem Set # 7

Starred (*) problems due Thursday, April 5

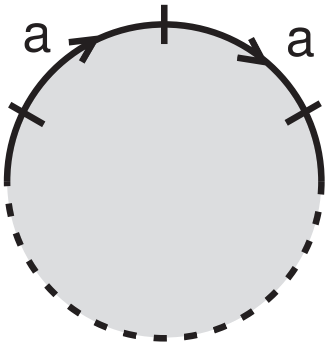
(*) 33. Find examples of spaces and subspaces $A_0 \subseteq X_0$ and $A_1 \subseteq X_1$ so that $H_*(X_0) \cong H_*(X_1)$ and $H_*(A_0) \cong H_*(A_1)$, but $H_*(X_0, A_0) \not\cong H_*(X_1, A_1)$. (If you want to make it more challenging, find examples with all of the spaces path-connected? Note that Problem #35 gives a hint on how <u>not</u> to solve this problem...)

34. Show that if $A \subseteq X$ and the identity map $I : X \to X$ is homotopic to a map $f : X \to X$ with $f(X) \subseteq A$, then for every n, $H_n(A) \cong H_n(X) \oplus H_{n+1}(X, A)$.
(So A has more "holes" than X does...)

35. (a): Let $f : (X, A) \to (Y, B)$ be a map of pairs such that both $f : X \to Y$ and $f : A \to B$ are homotopy equivalences. Show that the induced map $f_* : H_n(X, A) \to H_n(Y, B)$ is an isomorphism for all n.

(b): Show that the inclusion map $\iota : (D^n, \partial D^n) \to (D^n, D^n \setminus \{0\})$ satisfies the hypotheses of (a), but is <u>not</u> a *homotopy of pairs*, that is, there is <u>not</u> a map $f : (D^n, D^n \setminus \{0\}) \to (D^n, \partial D^n)$ so that $f \circ \iota$ and $\iota \circ f$ are both homotopic, as maps of pairs, to the identity maps.

36. Compute the singular homology groups of the pseudo-projective planes P_n, $n \geq 2$, shown below, where the boundary has been subdivided into n equal arcs.



(*) 37. For a space X the *cone* on X is the quotient space $cX = X \times I/\{(x, 0) \sim (y, 0) : x, y \in X\} = X \times I/X \times \{0\}$, and the *suspension* of X is the quotient space $SX = X \times I/\{(x, 0) \sim (y, 0), (x, 1) \sim (y, 1) : x, y \in X\}$, which can be thought of as two cones on X glued along their common copy of X. Show that for any path connected space X, $\widetilde{H}_i(cX) = 0$ and $\widetilde{H}_i(SX) \cong \widetilde{H}_{i-1}(X)$ for all i.

38. Show that, for any collection of finitely generated abelian groups $G_1, \ldots, G_n$, there is a path-connected space X with $\widetilde{H}_i(X) \cong G_i$ for all $i = 1, \ldots, n$ and $\widetilde{H}_i(X) = 0$ for all other i.