Math 871 Problem Set 9

Starred (*) problems are due Thursday, November 13.

(*) 57. Show that if X is limit point compact, and A is a closed subset of X, then A is limit point compact.

58. [Munkres, p.194, #12] Show that if $f : (X$
$mt) \to (Y, \mathcal{T}')$ is continuous and *open* and X is first countable, then $(f(X), \mathcal{T}'_{f(X)})$ is first countable.

(*) 59. [Munkres, p.194, #2]Show that if $(X, \mathcal{T})$ is second countable (with countable basis $\mathcal{B}$), then for <u>every</u> basis $\mathcal{C}$ with $\mathcal{T}(\mathcal{C}) = \mathcal{T}$ there is a *countable* basis $\mathcal{C}' \subseteq \mathcal{C}$ with $\mathcal{T}(\mathcal{C}') = \mathcal{T}$.

[Hint: look at all $C \in \mathcal{C}$ with $B_1 \subseteq C \subseteq B_2$ for some $B_1, B_2 \in \mathcal{B}$, and pick some of those....]

60. [Munkres, p.194, #5(a),#6] Show that if $(X, \mathcal{T})$ is a *metrizable*, separable space, then X is second countable. Conclude that $\mathbb{R}$ with the lower limit topology is <u>not</u> metrizable.

61. A space $(X, \mathcal{T})$ is called *Lindelöf* if every open covering $\{U_\alpha\}_{\alpha \in I} \subseteq \mathcal{T}$ has a *countable* subcover(ing). Show that a closed subset of a Lindelöf space is Lindelöf.

62. [Munkres, p.205, #1] Show that a closed subset $A \subseteq X$ of a normal (= T_1 plus T_4) space $(X, \mathcal{T})$ is normal.

(*) 63. Show by example that if $\mathcal{T}' \subseteq \mathcal{T}$ are topologies on X and $(X, \mathcal{T})$ is regular (= T_3 plus T_1), we cannot conclude that $(X, \mathcal{T}')$ is regular, even if it is T_1. Conclude that the continuous image of a regular space need not be regular. [Cheap route: think discrete topology...]

64. [Munkres, p.205, #2] Show that if $(X_\alpha, \mathcal{T}_\alpha)$ are <u>non</u>-<u>empty</u> (!) spaces and $\prod_\alpha X_\alpha$ is normal (in the product topology), then each X_α is normal. [Remember, though, the converse is false!]