Math 871 Problem Set 10

Starred (*) problems are due Thursday, December 4.

65. [Munkres, p.330, Problem #3(c)(d)] Show that if X is a contractible space, then any two maps $f, g : Y \to X$ are homotopic. Show in addition that if Y is path connected, then any two maps $f, g : X \to Y$ are homotopic.

(*) 66. The *cone* cX on a space X is the quotient space $X \times I/\sim$ where $(x,t) \sim (y,s)$ if $(x,t) = (y,s)$ or $t = s = 1$. (That is, we crush $X \times \{1\}$ to a point.) Show that for any space X, cX is a contractible space.

[One approach: cX can be viewed as a certain mapping cylinder.]

(*) 67. [Hatcher, p.18, Problem #3(c)] Show that if maps $f, g : X \to Y$ are homotopic and f is a homotopy equivalence, then g is also a homotopy equivalence.

68. [Hatcher, p.19, Problem #11] Show that if $f : X \to Y$ is a map so that there are maps $g, h : Y \to X$ with $f \circ g \simeq \mathrm{Id}_Y$ and $h \circ f \simeq \mathrm{Id}_X$, then f is a homotopy equivalence.

69. Show tht if $f, g : X \to Y$ and $h, k : Z \to W$ satisfy $f \simeq g$ and $h \simeq k$, then $f \times h, g \times k : X \times Z \to Y \times W$ are homotopic.

(*) 70. Show that if X is Hausdorff and $A \subseteq X$ is a retract of X, then A is a closed subset of X.

[Hint: Problem #40 of Problem Set 6 is useful here... find functions $f, g : X \to X$ so that A is the set of points where the functions agree!]

71. [Munkres, p.186, Problem #11a] Show that if $p : X \to Y$ is a quotient map, and Z is a locally compact Hausdorff space, then $p \times \mathrm{Id}_Z : X \times Z \to Y \times Z$ is also a quotient map.

[See the very detailed hints in Munkres' text!]