Math 856 Homework 5

Starred (*) problems to be handed in Thursday, November 9

(*) 25: Show that in the definition of a bundle map $(F, f) : (p_1, E_1, B_1) \to (p_2, E_2, B_2)$, the continuity of $F : E_1 \to E_2$ implies the continuity of $f : B_1 \to B_2$.

26: (b) Show that Whtiney sum "$\oplus$" is commutative and associative (up to bundle isomorphism), and $\varepsilon^n \oplus \varepsilon^m \cong \varepsilon^{n+m}$.

(a) Given two smooth manifolds M^n and N^m, if M admits a nowhere-zero vector field and $TM \oplus \varepsilon^1$ and $TN \oplus \varepsilon^1$ are both trivial, show that $M \times N$ is parallelizable. [Here ε^k represents the trivial k-bundle.]

(c) Show that $TS^n \oplus \varepsilon^1 \cong \varepsilon^{n+1}$. [Consider $T(S^n \times (-1, 1)) \subseteq T\mathbb{R}^{n+1}$.]

(d) Conclude that a product $S^{n_1} \times \cdots \times S^{n_k}$ of spheres is parallelizable, provided at least one of the n_i is odd.

(*) 27: If ξ is a vector bundle, show that $\xi \oplus \xi$ is orientable.

28: If M is a smooth manifold, show that TM is the trivial bundle $\Leftrightarrow$ T^*M is (without using our Riemannian metrics argument).

29: Let $X_1, \ldots, X_k$ be linearly independent vector fields on a manifold M with Riemannian metric $\langle \cdot, \cdot \rangle$. Show that the Gram-Schmidt procedure can be applied to the vector fields all at once, to give a collection of orthonormal vector fields $Y_1, \ldots, Y_k$.