Math 445 Homework 8

Due Friday, November 14

29. Find the continued fraction expansions of the rational numbers

$$191/73 \qquad\qquad \text{and} \qquad\qquad 112/53$$

30. [NZM, p.327, Problem 7.2.5] Show that if $x = [a_0, \ldots, a_n, b]$ and $x = [a_0, \ldots, a_n, c]$ with $b < c$, then $x < y$ if n is odd, and $x > y$ is n is even.

31. Find the continued fraction expansion of $\sqrt{29}$, and use this to find the first five (5) convergents of $\sqrt{29}$.

32. Show that for n a positive integer that is not a perfect square (translation: the continued fraction expansion of $\sqrt{n}$ never terminates), that at <u>every</u> stage of the continued fraction expansion of $x = \sqrt{n}$

$$x = \langle a_0, a_1, \ldots, a_{k-1}, a_k + r_k \rangle$$

r_k is always of the form $r_k = \dfrac{\sqrt{n} - a}{b}$, where $b | n - a^2$. Conclude that the continued fraction expansion of $\sqrt{n}$ will eventually repeat, with a period of length at most $n\lfloor\sqrt{n}\rfloor$.

Hint: by induction! In the inductive step, write $\dfrac{b}{\sqrt{n} - a} = \dfrac{\sqrt{n} + a}{c}$, and then find the fractional part of this. For the second half, how long must you wait before the r_k's *must* repeat themselves?