Math 445 Homework 7

Due Friday, November 7

25. Show that if p is an odd prime and a is a primitive root mod p, then $\left(\frac{a}{p}\right) = -1$.

26. The primes p for which $x^2 \equiv 7 \pmod{p}$ has solutions consists precisely of those primes lying in certain congruence classes mod 28 ; which ones?

    [Hint: if you think of classes mod 7 as being represented by $-3, \ldots, 0, \ldots, 3$ then you can recycle a lot of your work....]

27. Compute the (Jacobi) symbols $\left(\frac{31}{113}\right)$ and $\left(\frac{131}{311}\right)$.

28. [NZM, p.137, # 19] Show that for every (odd) prime p, the residue equation

    $$x^8 \equiv 16 \pmod{p}$$

    always has a solution.

    [Hint: $16 = 2^4$; look at our old criterion for solutions <u>and</u> our new ones for quadratic residues...]