Math 445 Homework 6

Due Friday, October 17

21. Show that if an integer n can be expressed as the sum of the squares of two *rational* numbers
$$n = (\frac{a}{b})^2 + (\frac{c}{d})^2 \ ,$$
then n can be expressed as the sum of the squares of two *integers*.

    (Hint: Not directly! Show that n has the correct prime factorization....)

22. [NZM, p. 106, # 2.8.8] Determine how many solutions (mod 17) each of the following congruence equations has:

    (a) $x^{12} \equiv 16 \pmod{17}$ (b) $x^{48} \equiv 9 \pmod{17}$
    (c) $x^{20} \equiv 13 \pmod{17}$ (d) $x^{11} \equiv 9 \pmod{17}$

23. If p is a prime, and $p \equiv 3 \pmod 4$, show that the congruence equation
$x^4 \equiv a \pmod p$ has a solution $\Leftrightarrow$ $x^2 \equiv a \pmod p$ does.

    On the other hand, show (by example) that if $p \equiv 1 \pmod 4$ this result need <u>not</u> be true.

24. [NZM, p.106, # 2.8.13] Show that, for a prime p, the numbers $1^k, 2^k, \ldots (p-1)^k$ are all <u>distinct</u> mod p $\Leftrightarrow$ $(k, p-1) = 1$.