Math 445 Homework 1

Due Wednesday, Sept. 10

1. (NZM, Problem 1.3.27) Show that if n is *not* prime, then $n|(n-1)!$.

2. (NZM, Problem 1.3.31) Show that if $f(x)$ is a non-constant polynomial with integer coefficients, then $f(n)$ cannot be prime for every $n \in \mathbb{N}$.
   (Hint: If $f(n) = p$ is prime, show that for every $k \in \mathbb{N}$ we have $p|f(n+kp)$; eventually $f(n+kp)$ is too big to be p ...)

3. (NZM, Problem 1.3.33) Show that for $n > 1$, $n^4 + n^2 + 1$ is *never* prime.
   (Hint: $f(x) = x^4 + x^2 + 1$ can be expressed as a product of quadratics; find the factorization!)

4. Show that if $2^n - 1$ is prime, then n must be prime.