Math 445 Homework 7

Due Wednesday, November 3

31. Find the continued fraction expansions of the rational numbers

$$53/18 \qquad\qquad \text{and} \qquad\qquad 115/53$$

32. [NZM, p.327, Problem 7.2.5] Show that if $x = [a_0, \ldots, a_n, b]$ and $x = [a_0, \ldots, a_n, c]$ with $b < c$, then $x < y$ if n is odd, and $x > y$ is n is even.

    [Hint: induction!]

33. Find the continued fraction expansion of $\sqrt{17}$, and use this to find the first five (5) convergents of $\sqrt{17}$.

34. Repeat problem # 33, for $\sqrt{19}$.

35. [NZM, p.336, Problem 7.5.3 (sort of)] If $\alpha < \beta < \gamma$ are irrational numbers, $\alpha = [a_0, a_1 \ldots]$, $\beta = [b_0, b_1, \ldots]$, $\gamma = [c_0, c_1, \ldots ...]$, and $a_i = c_i$ for $0 \le i \le n$, then $a_i = b_i = c_i$ for $0 \le i \le n$.

    [Hint: Induction! Use $\alpha = [a_0, \ldots, a_{i-1}, a_i + x_i]$, etc. and Problem #32 to compare $a_{i+1} = \lfloor \frac{1}{x_i} \rfloor$, etc. Note that if $x < y$ then $\lfloor x \rfloor \le \lfloor y \rfloor$.]